VISUAL COMPARISON OF PATTERNS: THE SIGNIFICANCE OF PATTERN TRANSFORMATION AND PATTERN ARRANGEMENT IN THE VISUAL FIELD.

by

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TITLE. Visual comparison of patterns: the significance of pattern transformation and pattern arrangement in the visual field.

I certify:

(i) That the greater portion of the work submitted in the above thesis has been done subsequent to my registration for the degree of Ph.D.

(ii) That the above thesis is my own account of my own research.

(iii) That no part of the work incorporated in the above thesis has previously been incorporated in a thesis submitted by me for a Higher Degree at any university.

Date 11th August, 1980 Signature

AFT/GMA

...What I tell you three times is true...

I said it in Hebrew - I said it in Dutch -
I said it in German and Greek:
But I wholly forgot (and it vexes me much)
That English is what you speak.

Lewis Carroll.
The Hunting of the Snark.

A B S T R A C T

The two main types of scheme for the internal representation and internal processing of visual stimuli involve either a 'pointillistic' internal representation to which various internal compensatory transformations can be applied, or a structural internal representation that consists of elements representing the local features in the stimulus and elements representing the spatial relations between these local features. The two types of scheme naturally predict different effects of stimulus arrangement on performance for the 'same-different' comparison of patterns. The results of a number of experiments designed to examine these effects show that:

(i) judgements of the 'sameness' of pairs of identical patterns are most affected by the *distance* between the two patterns;

(ii) judgements of the 'sameness' of pairs of patterns where one has been reflected or rotated through 180° are most affected by the *symmetry* of the pattern positions with respect to the point of fixation.

Since neither of the two types of scheme described above can adequately explain these results, a new scheme is proposed. In this scheme, the internal representation consists of elements representing the local features in the stimulus, elements representing the spatial relations between the local features, and additional elements representing the position of the stimulus with respect to the point of fixation. It is proposed that two types of operation can be performed on this internal representation: a global relabelling of element types, and a progressive continuous modification of individual elements.

The proposed scheme is shown to be adequate to explain the effects of pattern arrangement described above, and various predictions are made from the scheme, concerning the 'same-different' comparison of patterns related by reflections and by various other transformations. These predictions are verified in a number of further experiments which confirm all the major components of the proposed scheme.

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1. INTRODUCTION

1.1 The internal representation of visual stimuli

Given the vast number of receptors in the retina, and the vast number of distinguishable light levels which each of these can report, the number of patterns which could be discriminated using the retina as a detector is many orders of magnitude greater than the number of synapses in the brain. Further, since the eye is always moving in the world, and objects in the world are always changing, it is most unlikely that the eye will ever be stimulated in exactly the same way twice (Sutherland, 1968). Fortunately, given a knowledge of the types of object that exist in the world, and especially of those of importance to the organism, a large proportion of the information incident on the retina is redundant, and can be lost to the remainder of the visual system without loss of perceptual ability. It is a trivial task for humans to decide whether or not two visual stimuli correspond to the same object in the external world, and it is therefore natural to ask what information has to be discarded by the visual system before this decision can be made. Given the large amount of information which must be lost, it may be more appropriate to ask in what form is visual information *retained* by the visual system. This retained information is variously referred to in the literature as the 'Internal Representation', the 'Image' and the 'Stimulus Encoding'. The nature of the internal representation (hereafter referred to as the IR) is a topic of fundamental interest to researchers in visual pattern recognition, and, as will be seen, much effort has been involved in the derivation of models of the IR and of the internal processes that can be performed on it.

In the present study, some of these models are evaluated empirically. The evaluation is made using the results of a number of experiments in which subjects made visual comparisons of pairs of patterns which were related by various transformations and which were presented in various arrangements in the visual field. It will be suggested that the outcome of these experiments is incompatible with the two main types of model that are at present popular, and a new model will therefore be put forward and tested.

This type of experimental and theoretical framework is currently the subject of a controversy (Kosslyn & Pomerantz, 1977; Anderson, 1978, 1979; Hayes-Roth, 1979; Pylyshin, 1979a). The debate concerns whether the IR is 'pictorial' or 'propositional' (for a definition of these terms, see below), and whether it is possible to distinguish between these alternatives by behavioural experiment. Anderson maintains that the distinction cannot be made, for, given any form of IR (propositional or pictorial) and a set of legal operations which may be performed on it, one may always postulate a second form of IR (pictorial or propositional respectively), together with suitably modified operations, which completely mimics the behaviour of the first. Pylyshin rejects this argument on the grounds of the 'explanatory adequacy' and 'predictive power' of a given model, and Hayes-Roth rejects the argument on the grounds that a good model is *vulnerable* to experimental disproof. With these arguments in mind, the models discussed in this study are evaluated using the following criteria. A model is acceptable if it has predictive power; if it is parsimonious; if it has computational utility; if it accounts for at least some of the known properties of the visual system. Predictive power is perhaps the vaguest notion; most models are unspecific about the performance one should expect if they were in operation. The strategy adopted here is to make the simplest and most intuitive assumptions possible, and to determine whether the model together with the assumptions naturally predicts known behaviour, and whether it can make *a priori* predictions about behaviour. Using these criteria it should be possible to make the determination between 'pictorial' and 'propositional'.

In the remainder of this chapter, the two main types of model for the IR ('propositional' and 'pictorial') will be described, and evidence concerning the validity of each will be presented. Certain results which are not predicted by either type of model will be described, and finally an outline of the rest of the study will be presented.

1.2 Models of the internal representation and the processes which can act on it

The types of scheme referred to above as 'pictorial' and 'propositional' are often referred to as 'transformation' and 'structural' schemes respectively. In transformation schemes (Pitts & McCulloch, 1947; Marko, 1973; Foster & Mason, 1979) the IR is usually regarded as 'pointillistic'. That is, the IR preserves at least some of the spatial or metric properties of the stimulus in its own spatial or metric structure. This type of IR is sometimes referred to as a 'template'. For the purpose of comparison, the IR can be modified by certain internal compensatory transformations such as, for example, translations, dilatations, and in some schemes, rotations.

In structural schemes (Sutherland, 1968, 1973; Barlow, Narasimhan &

Rosenfeld, 1972; Foster & Mason, 1979) the IR is thought to specify certain local features in the stimulus, and the spatial relations obtaining between these features. A local feature might be an edge or a corner and a spatial relation might specify that one feature is, for example, 'left of' another, or 'joined to', or 'above'. The similarity of stimuli is determined by the extent to which their structural descriptions concur.

It is possible to postulate models in which, for example, compensatory transforms are applied to a structural IR. However such notions do not always make sense. For instance there is no meaning to rotating an encoding which takes account of purely topological features of the stimulus. In general, transformations (at least of the geometrical type) are thought to be associated with holistic, pictorial or pointillistic representations; the few operations which have been postulated to act on structural representations are simple global relabellings of relations (Foster & Mason, 1979).

1.2.1 Transformation schemes

Transformation schemes were the first schemes postulated to explain the visual system's ability to recognize objects in the external world. Given that the *identity* of an object is invariant under a number of transformations, it was suggested that the visual system averaged functions of the 'distribution of excitation' as the latter was transformed by all members of a group of transformations (Pitts & McCulloch, 1947). This average was then a 'universal', and would be the same for all suitably transformed versions of the stimulus presented to the organism. Recognition based on the universals would automatically be invariant to the transformations in the group.

In a more recent scheme, transformed IRs are thought to be matched to prototypes (Marko, 1973). A range of compensatory transformations including dilatations, rotations and translations, defined by six parameters, are applied to the IR. The IR is then matched to object prototypes stored in memory, and the six parameters are adjusted to optimize the match. The prototype giving the best match defines the identity of the object.

A different type of transformation scheme has been suggested to explain 'mental rotation' effects (Shepard & Metzler, 1971; Cooper & Shepard, 1973a; Cooper, 1975; Cavanagh, 1977; Hock & Tromley, 1978). In experiments concerning these effects, subjects are asked to decide if rotated stimuli are reflected or standard versions of stimuli which are, for example, well known (letters), learnt in a single orientation prior to the experiment (nonsense forms), or presented side by side (3-dimensional block figures). The time taken to respond (reaction time) is found to be linearly related to the angle of rotation, up to 180° (see Fig. 1.1), except in the case of letters, where the relationship is monotonic but non-linear. From these and related experiments, the deduction is made that the subject is performing a 'mental rotation' of a 'mental image' of the stimulus; that this rotation has a measurable velocity; and that the 'image' passes through intermediate stages of rotation before reaching its final state. There are some problems with this interpretation which will be discussed later. Here it is worthwhile simply to note that the subject is not being asked to make judgements about the identity of the stimulus; his task is more akin to mental arithmetic or chess playing than to recognition (Chase & Simon, 1973; Cooper & Shepard, 1973a). Further studies suggest that subjects can mentally rotate images in preparation for a task, and that performance on these images is almost as good as performance on real rotated stimuli (Cooper & Shepard, 1973b).

Mental transformation is not limited to rotations; similar effects have been found for size scaling (Besner & Coltheart, 1976; Larsen & Bundesen, 1978) and for scaling in general (Dixon & Just, 1978). In all of these mental transformation studies the emphasis is on the process, not the IR. However, the notion of mental rotation only makes sense if it is viewed as a shape-preserving mapping, which must therefore act on a shape-preserving IR (Cooper, 1975).

1.2.2 Structural schemes

In Section 1.1 it was suggested that it is necessary for the visual system to discard some of the information incident on the retina before recognition or comparison of visual stimuli can proceed. One of the drawbacks of transformation schemes is that they contain no implicit mechanism which can take advantage of redundancy in the stimulus. That is to say that transformation schemes take no account of the meaning or structure within the stimulus; the pictorial nature of the IR means that, in principle, all stimuli should be recognized equally well. Often, however, this is not the case. For example, the ability to recognise visual stimuli can depend on the degree of homogeneity of the set from which the stimuli are drawn (Goldstein & Chance, 1971); the ability to discriminate visual stimuli can depend on the presence of *redundant* picture elements (Pomerantz, 1978); the ability of chess masters to recall chess positions depends crucially on the significance of the positions, in the sense that chess masters can recall positions from real chess games far better than they can recall random positions (Chase & Simon, 1973).

One way of making use of the structure in the world is to specify the types of local features of which objects in the world are commonly constructed, and to specify the way in which these features can usually be related to one another. A stimulus can then be represented by a set of elements associated with the features in the stimulus and further elements specifying the relations between these features. Schemes which operate in this way are known as '*structural*' or '*feature-relation*' schemes (Sutherland, 1968, 1973; Barlow, Narasimhan & Rosenfeld, 1972; Foster, 1978; Foster & Mason, 1979). Suitable features might be, for example, 'lines', 'blobs', 'curves', 'edges' or 'corners', and suitable relations might be, for example, 'left of', 'above', 'inside', 'joined to' or 'near'. The similarity of two stimuli is evaluated by the extent to which their structural descriptions concur.

Structural schemes have great computational utility, since the IR discards much of the redundant information in the stimulus. In contrast, any pictorial or pointillistic representation must contain a large number of picture points which contain very little information whatsoever (Barlow, Narasimhan & Rosenfeld, 1972). Apart from the economy of representation provided by structural schemes, they are capable of explaining various experimental observations which are not compatible with transformational schemes. When subjects make 'same-different' comparisons of pairs of tachistoscopically presented patterns related by rotations, performance falls off as the angle of rotation approaches 90°, and then rises again in the region of 180° (Dearborn, 1899; Rock, 1973; Foster, 1978), (see Fig. 1.2). If the simple operation of globally relabelling all relations by their opposites is applied to a feature-relation representation, inversion of the IR is trivial, and the elevation in performance at 180° is explained. A transformation scheme endowed ad hoc with an inversion operator will not predict the precise form of the dependence of performance on angle of rotation, whereas a feature relation scheme endowed with such an operator fits the data remarkably well (Foster & Mason, 1979).

The discrete nature of the IR in structural schemes implies some deficit in the veridicality of perception, because an infinite number of stimuli are represented by a finite number of elements. For example, a line of three dots might be represented as 'linear', 'slightly curved', or 'curved', with no other possibilities. In experiments on the discriminability of this sort of stimuli, Foster (1979) has shown that discrimination performance shows precisely the properties of such a discrete representation.

1.2.3 What is rotated in 'mental rotation'?

In the literature, studies on 'mental rotation' seem in general to be considered separately from studies concerned with structural schemes. This may not be inappropriate. Mental transformation tasks usually involve questions such as whether a stimulus is a standard or reflected version; or whether a stimulus is symmetrical or asymmetrical. Observers are usually highly trained; the stimuli are usually overlearnt and very simple; inspection of the stimuli is allowed; and the time taken to respond can be as long as five or ten seconds.

Conditions are very different in most experiments concerned with the ability of subjects to recognize pairs of patterns related by 180°

rotation. The task involves questions such as whether two stimuli appear 'same' or 'different'. The observers are naive and receive no training; the stimuli are unfamiliar and are seen only once; presentations are tachistoscopic, precluding directed eye-movements; the time taken to respond is typically less than one second. Under such disparate conditions it is not surprising that the results of the two types of experiment are so different; it should be possible to reconcile the two processes.

Cooper (1975) presents a study in which she claims to refute the notion that the 'mental image' is structural, because the rate of mental rotation does not depend on a particular measure of the complexity of the stimuli which she uses. Elsewhere, however, it has been shown that the rate of 'rotation' depends on the nature of a (non-rotated) matching stimulus (Pylyshin, 1979b) and on the 'landmarks' within a (rotated) stimulus (Hochberg & Gellman, 1977). Different 'rotation' tasks give rise to different rates of 'rotation' (compare the rates reported in Shepard & Metzler (1971) with those reported in Cooper (1975)). Clearly 'mental rotation' does depend on structural elements both in the 'rotated' stimulus and in the standard stimulus - an effect which is not compatible with a 'template' IR.

In a three-dimensional mental rotation task similar to that used in Shepard and Metzler's original study, it has been shown that the total number of eye movements between the two stimuli (which were presented side-by-side) depends linearly on the angle of rotation relating the stimuli (Just & Carpenter, 1976). If the subject were 'rotating' a holistic 'image', one would not predict that, after rotating the image through some angle (say, 50°), the subject would need to refresh the image with a non-rotated stimulus. This particular result seems far more compatible with a structural IR in which the observer is searching for a feature-relation which will tell him whether the two stimuli are 'same' or 'reflected'; that is, the mental rotation is being performed on individual structural elements, not on holistic 'templates'.

Structural schemes do not naturally predict the results of Shepard & Metzler et al, since they make no commitment about the extraction of information about the 'sense' of a stimulus (i.e. whether it is 'standard' or 'reflected'). If, however, the IR were structural, and the system were not specifically equipped to perform rotations, subjects would have to mentally rotate elements in the IR, one at a time, and they would have to learn to use the rotated elements to make the relevant decision. This may explain the long time taken to perform 'mental rotations'. The ability to perform this task is not innate any more than playing chess is innate; the ability to memorize chess positions is something that must be learnt (Chase & Simon, 1973). Similarly mental rotation must be learnt and therefore cannot be performed rapidly. In contrast, the ability to discriminate 'same' inverted pattern pairs from 'different' pairs is apparent in untrained, naive subjects, and the time taken to perform the task is relatively short.

Hence, we conclude that 'mental rotation' is more likely to be mediated by a structural IR than by a pictorial IR, and that the rotation is performed on elements of the IR, not on the IR as a whole. This resolves the apparent contradiction between the results of the two different paradigms.

1.3 The importance of stimulus configuration in the visual field

So far we have concentrated on the way in which visual discrimination and identification performance is affected by simple transformations of stimuli. In most of the experiments described above, the pairs of stimuli were presented either simultaneously, side-by-side, or sequentially, both in the same central position. It is usually assumed that the geometries of these two types of presentation are perceptually equivalent, and that in general the arrangement of patterns in the visual field is not important. There is evidence, however, that the arrangement of the patterns does have effects on performance, and that these effects interact with the effects of transformations. As a simple demonstration, consider the triangles in Fig. 1.3 (after Attneave, 1950). When asked to decide which two triangles are more similar, most observers will chose the lower pair, despite the fact that the triangle on the lower left is identical to the triangle on the upper right, and vice versa. So perceptual similarity depends on the arrangement of the stimuli. Further examples of the importance of stimulus arrangement follow.

1.3.1 Discrimination of mirror images and detection of symmetry

When children are asked to discriminate mirror image pairs from repeated pairs, they make more horizontal mirror confusions when the stimuli are side-by-side and more vertical errors when the stimuli are one above the other (Sekuler and Rosenblith, 1964). A similar effect has been demonstrated for adults; when asked to discriminate mirror images presented side-by-side but with a vertical displacement of one

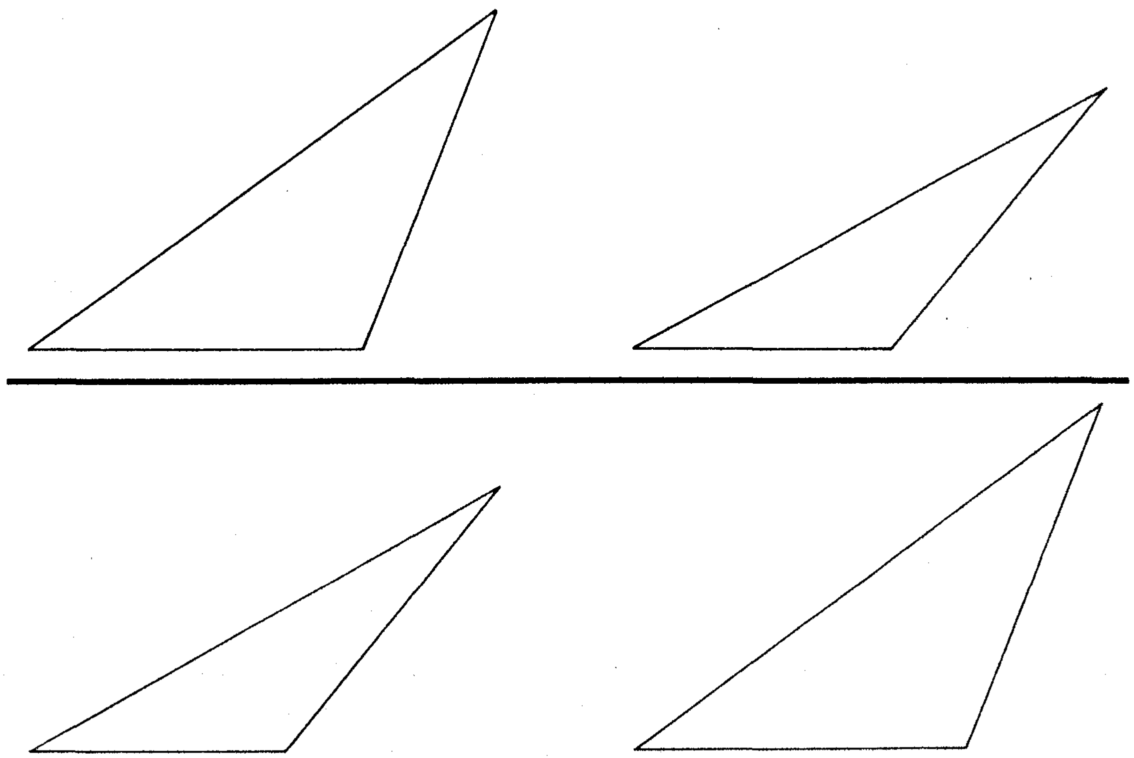


Fig 1.3

Figure 1.3. The effects of pattern arrangement. Subjects usually judge the lower pair of triangles to be more similar to each other than the upper pair, despite the fact that the lower left triangle is identical to the upper right and vice versa. *After* Attneave (1950).

of the images, reaction time to 'left-right' mirror pairs is not affected by the amount of vertical displacement, whereas reaction time to 'up-down' mirror pairs is increased by increasing vertical displacement (Sekuler & Pierce, 1973).

In 'same-different' judgement tasks the time taken to report the 'sameness' of mirror pairs is shorter when the patterns are presented symmetrically about the point of fixation than when they are both presented to one side (Corballis & Roldan, 1974; Bradshaw, Bradley & Patterson, 1976) and side-by-side mirror pairs are better discriminated from different pairs when the mirror axis is vertical rather than horizontal (Foster & Mason, 1979). Symmetry in a complex field of random dots is best perceived when the observer fixates a point on the axis of symmetry (Julesz, 1971; Barlow & Reeves, 1979).

From these studies we can deduce that the two patterns in a mirror pair appear in some sense most similar when two conditions hold.

(i) One pattern must be able to be taken into the other by a single reflection, that is, no translation should be involved. For example, when the patterns are side-by-side and related by a reflection in a horizontal axis, the simplest transformation which will map one pattern onto the other is a reflection combined with a horizontal translation. Under these conditions patterns appear less similar. In other words there must be an axis of symmetry in the display as a whole, to obtain maximum 'similarity'.

(ii) The point of fixation must lie on the mirror axis.

These effects are not naturally predicted by either transformation or structural schemes. Although structural schemes can predict the ease of detecting the sameness of reflected patterns, for example by inversion of all horizontal relations in the IR (Foster & Mason, 1979), they make no prediction about the importance of the position of the point of fixation or about the importance of the orientation of the mirror axis. We will return to this topic in Chapter 4.

1.3.2 Oblique effects

The orientation of the axis of symmetry of a pattern has been shown to affect the time taken to detect symmetry (Palmer & Hemenway, 1978). The response is fastest when the axis is vertical, slightly slower when the axis is horizontal, and much slower when the axis is oblique. This effect is a detection analogue of the well known oblique effect in the discrimination of the direction of stimuli (see, for example, Appelle, 1972; Attneave & Olson, 1967). Neither transformation nor structural schemes make any predictions about oblique effects.

1.3.3 The effects of dilatations

The study of the perceptual effects of the dilatation (or magnification) of one of a pair of stimuli can be divided into two parts; the study of 'size constancy', and the study of reaction times as a function of the size difference between stimuli.

'Size constancy' is a phenomenon in which the subject reports the true size of a stimulus rather than the size of its image on the retina, independent of the distance between the stimulus and the subject (Epstein & Park, 1963; Hochberg, 1972). This phenomenon occurs even under conditions of reduced cue information (Gogel, 1971; Liebowitz, Wilcox & Post, 1978). 'Size constancy' is not really concerned with the perception of the form of the stimulus, but rather with the perception of form-independent global attributes such as the size of the stimulus and its distance from the observer.

The study of reaction times as a function of size differences is often considered to be the size-scaling analogue of the mental rotation studies mentioned above, although the task is more often discrimination of 'same-different' than discrimination of 'same-reflected'. When subjects are asked to judge whether two stimuli of different sizes are 'same' or 'different' (other than in size), the time taken to respond is approximately linearly related to the ratio of the sizes of the stimuli, and this 'size scaling' can be performed independently of rotations, in the sense that the reaction time effects of rotation and magnification do not interact (Sekuler & Nash, 1972; Besner & Coltheart, 1976; Larsen & Bundesen, 1978).

Neither structural nor transformational schemes have any specific predictions about size scaling, although the linear increase in reaction time with size ratio would seem to favour an active scaling operation rather than the existence of the size independent IR which has been suggested in some structural schemes (for example, see Sutherland, 1968).

1.3.4 The perception of rotated figures

Although some authors talk of the invariance of form under rotation (see, for example, Deutsch, 1966), the difficulty associated with the recognition of rotated figures and with comparing these figures to standard non-rotated forms is well established (Dearborn, 1899; Arnoult, 1954; Gibson et al, 1962; Hake, 1966; Sutherland, 1968; Rock, 1973;

Attneave & Olson, 1967; Foster, 1978; Pomerantz, 1978; Wicklegren, 1979) although in most of these studies detection or recognition performance on the task was above chance level for all rotation angles, and, as mentioned above, performance is higher at rotation angles in the region of 180° than at those in the region of 90°. If one explains this elevation in performance at 180° by suggesting that all the relations are reversed in a structural IR (see section 1.2.2), one is left with the question of why performance on 90° rotated patterns is above chance level, since in theory there exists no mechanism for performing this task. Sutherland (1968) suggests that, in octupuses and goldfish, recognition of rotated shapes is facilitated by special features in the stimulus. Some evidence that this is true of human observers is presented in Chapter 6.

1.4 The purpose of the present study

Although some of the experiments discussed in the previous sections take account of the relationship between the point of visual fixation and the arrangement of stimuli in the field, there has been no systematic study of this topic. One reason for this might be the widespread acceptance of the classical notion of the invariance of form with respect to retinal position (see, for example, Sutherland, 1968, 1973). Most of the evidence in favour of this notion comes either from introspective reports or from animal behavioural studies. If a monkey is trained to behave in a certain way to a stimulus in a certain retinal position, the subsequent transfer of the behaviour to the same stimulus in a different retinal position is taken as evidence that the form of the stimulus is invariant to this change of position. Logically, however, one can only infer that the stimulus at the new position gives rise to a percept which is in some sense *similar* to the original percept; one cannot infer that the two percepts are *identical*.

This study explores the effects of the arrangement of pattern positions, positional symmetry, and the orientation of pairs of pattern positions, and how these factors affect the ability of subjects to perform simple 'same-different' discrimination tasks on pairs of patterns related by certain transformations. The aim of the study is to gain more insight into the nature of the IR and the processes which can operate on it. As will be seen, non-standard pattern arrangements produce some non-intuitive effects. A model of the IR and its processes is put forward to explain these effects. The model makes some compromise between the structural and transformational viewpoints. The model explains the results of the first set of experiments and makes certain predictions, some of which are verified in further experiments.

There follows an outline of the remaining chapters in the thesis.

Chapter 2. Details are given of the equipment and computer programs which were constructed for the experiments. The nature of the experimental tasks and paradigms is explained and the general theory of the experimental designs and data analyses is described.

Chapter 3. Four experiments are described. These explore the effects of positional symmetry and separation on the visual comparison of pairs of patterns which are related by reflections, rotations, or which are identical. The ability to detect the 'sameness' of identical patterns is shown to depend mainly on the distance between the patterns, whereas the ability to detect the 'sameness' of pairs of patterns related by reflection or rotation through 180° is shown to depend mainly on the symmetry of the positions of the patterns with respect to the point of fixation.

Chapter 4. The effects found in the previous chapter are shown not to be naturally predicted or explained by either transformation or structural schemes. A new theory is put forward, which is neither exclusively transformational nor structural; the theory is shown to be capable of explaining most of the results of Chapter 3, and certain specific predictions for further experiments are made from the theory.

Chapter 5. Experiments testing three predictions of Chapter 4 are presented. The predictions concern:

(i) the 'best' reflection axis for detecting 'sameness' of pairs of patterns related by reflections;

(ii) the expectation of an 'oblique' effect in the ability to detect 'sameness' in such pattern pairs;

(iii) the expectation of a constant ability to detect the 'sameness' of pairs of patterns related by reflection in a vertical line, independent of their vertical position relative to the point of fixation.

All three predictions are found to be correct; this gives strong support to the model presented in Chapter 4.

Chapter 6. An experiment is described in which certain pattern specific effects are investigated. The results are shown to be consistent with the notion that the ability to detect the "sameness" of pairs of patterns related by 90° rotation depends strongly on the existence of some pattern specific attributes, whereas the ability to detect the 'sameness' of pairs which are related by 180° rotation or which are identical is not strongly dependent on such attributes. It is suggested that the former type of pattern can be detected as 'same' only by use of an inefficient non-structural feature-matching process which requires special features for its operation, whereas the latter types of pattern can also be detected as 'same' by the more efficient structural-matching process described in Chapter 4, which is pattern independent.

Chapter 7. Two experiments are described. These investigate the effects of positional symmetry and separation on the ability to detect the 'sameness' of pairs of patterns related by dilatations. The results of these experiments are unexpected and are not explained by any of the models presented here; it is suggested that further research is necessary to resolve the issue of how size information is encoded in the IR.

Chapter 8. An experiment testing a prediction of Chapter 4 is presented. The prediction concerns the ability to detect the 'sameness' of pairs of patterns related by certain non-rigid transformations, and is shown to be correct.

Chapter 9. The arguments and evidence of the preceding chapters are summarized.

2. GENERAL EXPERIMENTAL METHODS

In this Chapter the general methods which were used in this study are outlined. Many of the sections concern matters common to all the experiments; the reader may, however, wish to skip some of these sections and refer to them as necessary.

In most of the experiments reported here, subjects viewed large numbers of pairs of briefly presented patterns of ten randomly positioned dots. One pattern in a pair was either a transformed version of the other (for example rotated, reflected, unchanged) which will be termed 'same', or the two patterns were unrelated or 'different'. In mathematical terms any *n* dot pattern can be regarded as a transformation of any other *n* dot pattern; here the term transformation will be used to refer exclusively to rotations in the plane, reflections about axes in the plane, translations and dilatations (that is, magnifications). Subjects were required to judge which pairs were 'same' pairs and which were 'different'. In some experiments the two patterns in a pair were presented simultaneously side-by-side; in the remainder they were presented sequentially.

2.1 Apparatus

The viewing system and the signal invertor mentioned below were constructed by the author. Also, the programs and software described in Section 2.2. were developed by the author.

All the experiments were run using a mini-computer to generate and display the stimuli, and to record, analyse and plot the results.

Fig. 2.1 shows the apparatus used in the experiments. Stimuli

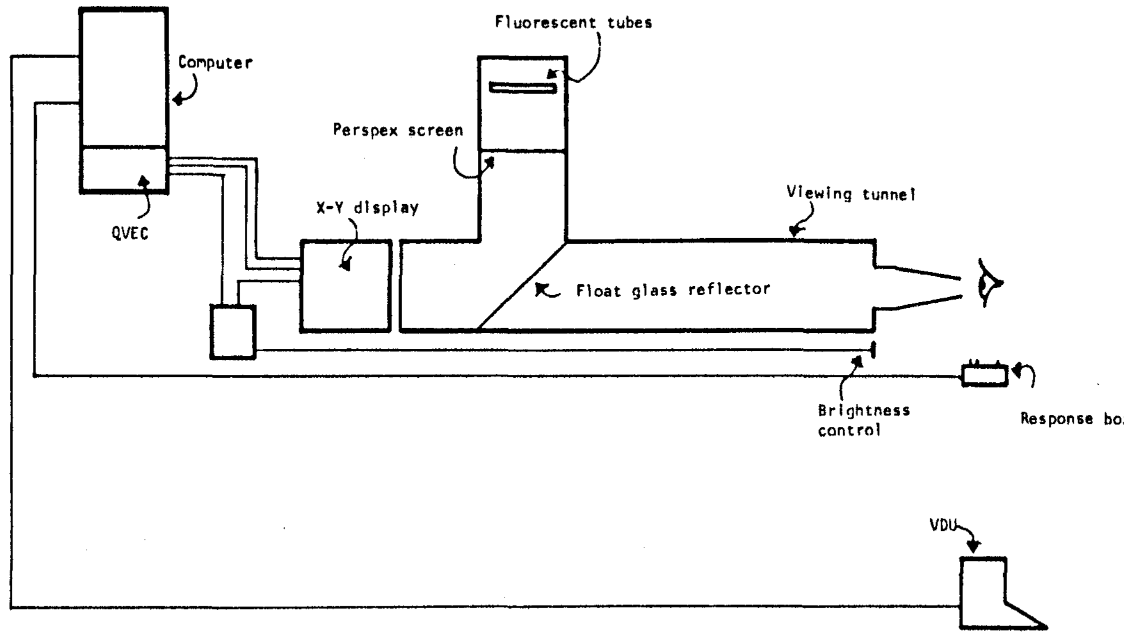


Fig 2.1

Figure 2.1. The apparatus used for the experiments.

were displayed on a Hewlett Packard 1300A X-Y display CRT. The phosphor (P4 sulphide) had a decay time of less than 1 msec.

A viewing system was used to avoid distractions and to control light levels. The system consisted of a tunnel through which the stimuli were seen superimposed on a white background field. The superimposition was achieved by the reflection of the field in a sheet of float glass. The dimensions of the background field were 22 cm. by 18 cm., and it was viewed from a distance of 170 cm., so that the field subtended about 7.4° by 6.2° at the eye. The light source for the background field consisted of four Thorn 'Daylight cool light' fluorescent lamps powered from a stabilized D.C. supply to avoid flicker. The light from the lamps was diffused by a sheet of white perspex, and a rectangular black mask directly in front of the perspex was seen reflected in the sheet of float glass at the same distance from the observer as the stimulus on the face of the display. The luminance of the background field was about 60 cd m^{-2}. To ensure that the area surrounding the background field was seen as totally black, masks were placed at intervals along the tunnel.

The subject's head was steadied by a viewing hood which was attached to the end of the tunnel.

As an aid to fixation and convergence, four small red light emitting diodes were arranged about the screen in a square whose side subtended 4° visual angle; the diodes were seen superimposed on the background field.

A signal invertor was used to correct the bright-up signal from the computer display generator which was of the wrong polarity. Incorporated in the invertor was a device which allowed the subject to set the luminance of the display remotely, while viewing it through the system.

The subject initiated trials and gave his responses using a hand-held push button box which was linked to the computer. Two buttons at the top of the box were used for 'yes' and 'no' responses, and one at the centre of the box was used for starting each trial.

The experimenter controlled the experiment from outside the laboratory using a visual display unit. The computer, which was a CAI Alpha, was remote from the laboratory. The stimuli were generated by a QVEC display driver which was interfaced to the computer via direct memory access. The QVEC could be programmed to draw points or vectors on the face of the display, and a very large number of these (about 10,000) could be displayed in one 20 msec frame.

2.2 Software

The computer was programmed to generate and store the coordinates for the stimuli, to present the stimuli and log the responses and response times, and to store these on disc at the end of each run. The computer was also programmed to analyse the data in various ways and to generate graphical output.

2.2.1 Random number generator. The random dot patterns and the balanced presentation sequences (which will be described below) were generated using a linear congruential sequence generator (Knuth, 1971). The sequence was defined as follows:

$$X_{n+1} = (aX_n + c) \bmod m$$

where X_n is the n^{th} random number,

$m = 10^9$

$a = 581754621$

and $c = 211324867$.

This sequence has the property that all the numbers in the range 0 to (10^9-1) occur in one cycle in pseudo-random order. Truncation of these to 6 significant figures yields a sequence of numbers with a flat distribution and very low serial correlation.

2.2.2 Typical experimental programs. A flowchart for a typical set of programs is shown in Fig. 2.2.

PG - "Pattern generator" This program generates the coordinates of the dots in the patterns, and the sequences controlling the order of the experimental treatments, and stores these on disc.

PR - "Pattern presenter" This program finds the stored sequences and pattern coordinates and loads them into memory; it then performs the required transformations on each set of pattern coordinates in turn; after the subject has initiated the trial by pressing the 'show' button it presents the patterns; it waits for a response which it records along with the response time; and finally, at the end of the run, it stores a record of all these transactions on disc.

UP - "Data file update" This program creates a data file and then, after a number of runs, it is used to collect all the output from the runs to make an updated summary of the raw data.

LO - "Look at raw data" This program is used to inspect or print out the data files created by UP. The data is in the form of the total number of correct 'same' responses and the total number of correct 'different' responses in each condition. (The program also displays average response times in case they are needed as an extra performance measure.)

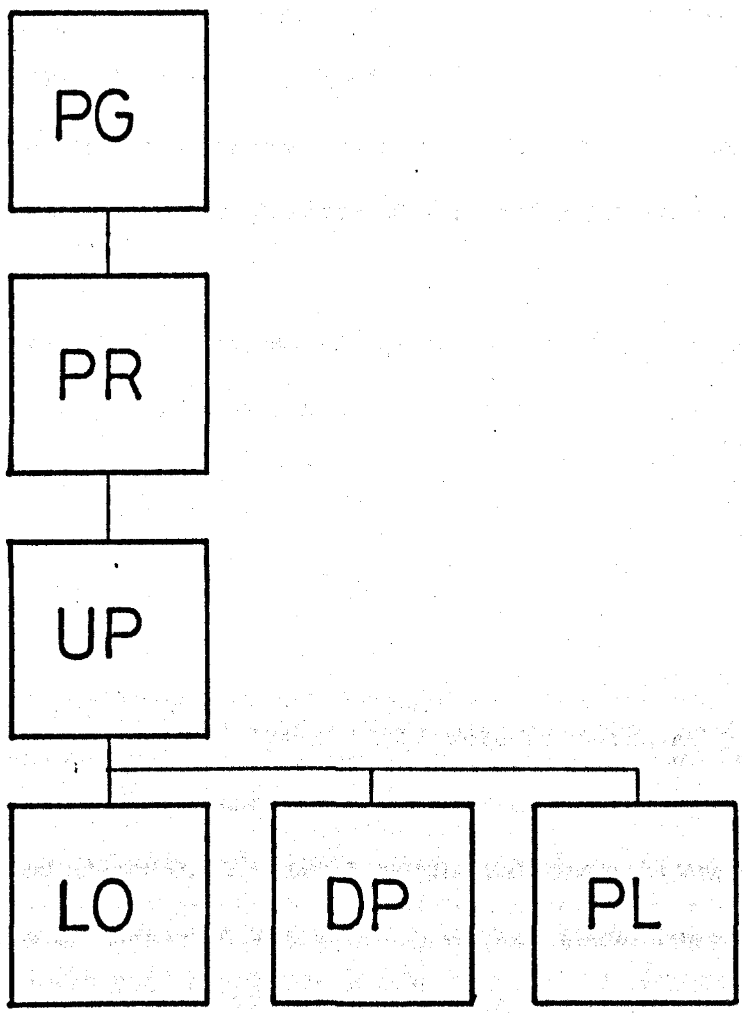


Fig 2.2

Figure 2.2. Flowchart for a typical set of programs used to control the experiments and to analyse the results.

DP - "d' analysis" This program analyses the data in terms of the discrimination index d' (see below), and can perform this analysis for individual subjects or as an average over all subjects.

PL - "Plot" This program is the same as DP but the data is presented in graphical form on a plotter.

All the experiments were run using some variant of this design. Other software was written to perform the statistical tests described below.

As the experimental programs all required the storage of large amounts of data, they all made use of a specially developed disc input/output package.

2.3 Stimuli

Except where stated, all the stimuli were patterns of ten randomly positioned dots generated within an imaginary disc of 0.5° diameter. Each dot subtended about 0.03° visual angle and the minimum centre to centre separation of the dots was 0.05°. Within these constraints the distribution of the dots was random. Some examples of such dot patterns are shown in Fig. 2.3, and a flow chart of the pattern generating program is shown in Fig. 2.4. No random dot patterns were used more than once, except in Experiment 6 where it was required to use a fixed set of patterns.

The stimuli were white and appeared superimposed on the white background field. At the start of each experimental session, the subject set the brightness of the dots to ten times increment luminance threshold, by adjusting the brightness of a flashing dot pattern viewed through a 1 log unit neutral density filter. This adjustment

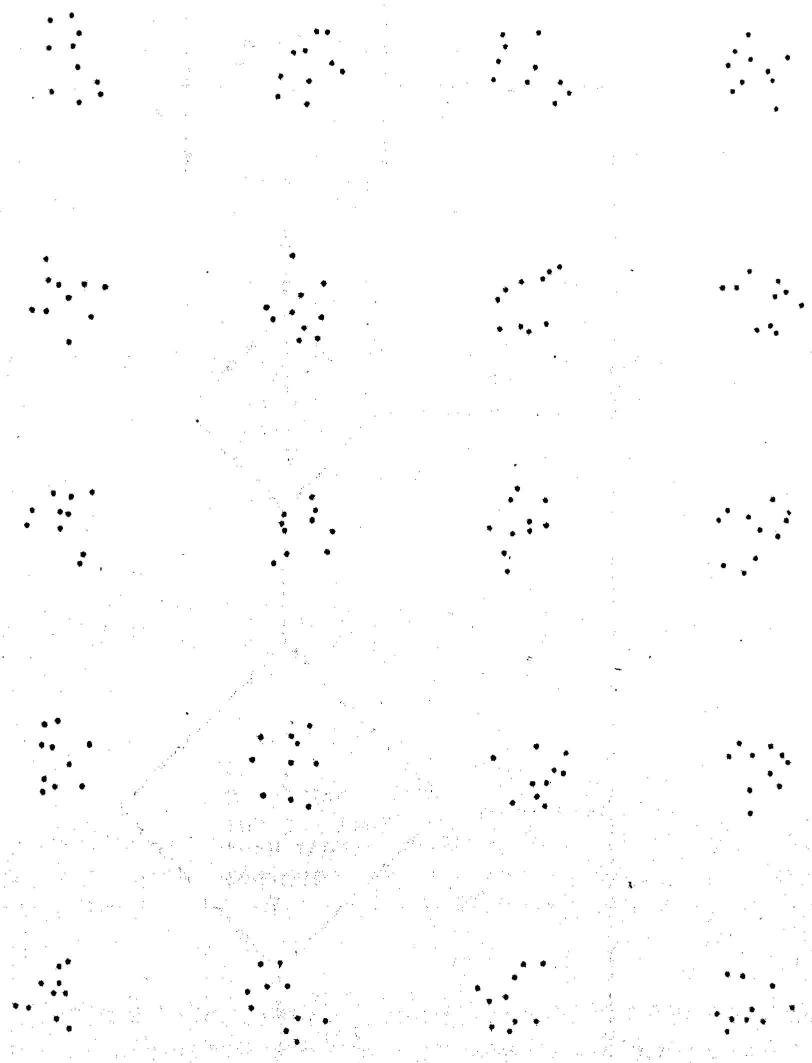


Fig 2.3

Figure 2.3. The stimuli. Some examples of the random dot patterns used as stimuli in the experiments. Not to scale. The stimuli appeared white on a white background. The diameter of the patterns was 0.5°.

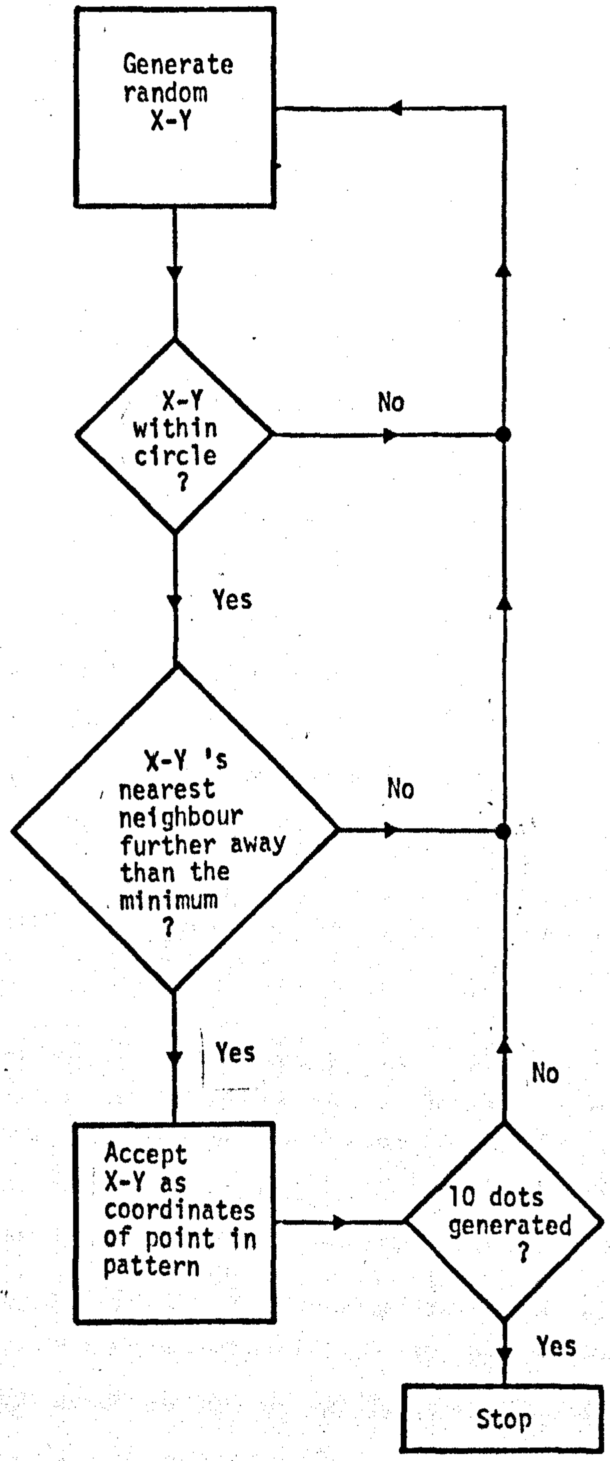


Fig 2.4

Figure 2.4. Flowchart for dot pattern generation.

was sufficient to ensure that the brightness of the dots was roughly constant over sessions for each observer.

Random dot patterns were chosen as stimuli since they are unfamiliar to the subject and thus have no properties such as meaning, name, and conventional handedness, which can be ascribed to, for example, letters and geometrical figures.

2.4 The experimental task and instructions

Subjects were presented with a series of pairs of patterns, and asked to judge, for each pair, whether the patterns were 'same', taking into account that one pattern may have been a transformed version of the other, or 'different'. Depending on the experiment the two patterns were presented simultaneously or sequentially. Instructions to the subject were typed on a card and the subject was told to ask any questions he wished if he felt he did not fully understand them. The instructions were designed to encourage the subject to respond reasonably rapidly, so that one could expect a degree of uniformity in response strategy over subjects. An example of the instructions follows.

> The subject will be presented with a number of pairs of random dot patterns. These pairs will be of two types:-
>
> (a) the two patterns are the same except that one pattern may have been moved in the plane of the screen and possibly transformed in some way with respect to the other (i.e. rotated or reflected); OR
>
> (b) the two patterns are totally unrelated.
>
> The subject is asked to decide if the patterns are SAME as

in (a) above. If so, the subject should respond 'yes' - if DIFFERENT, as in (b), the response is 'no'.

The subject will see a fixation pattern consisting of a partial cross with a dot in the centre. The subject should fixate the centre dot and, when ready, press the button marked 'show'. The fixation spot will then disappear. The pattern pair will briefly appear on the screen, one pattern after the other, after which the subject should respond by pressing the buttons marked 'yes' or 'no' to give his answer. The subject is asked to respond as quickly as possible whilst maintaining accuracy. When the fixation pattern reappears the system is ready for the next presentation.

The subject should FIXATE the centre of the partial cross FROM THE TIME HE INITIATES THE DISPLAY UNTIL THE PRESENTATION OF BOTH PATTERNS IS COMPLETE.

A square will appear when the run is complete.

The subject may rest as he wishes provided that the fixation dot is on the screen.

After reading the instructions the subject was given a trial run of ten to fifteen presentations to familiarize him with the timing of the display and with the use of the response box. No feedback was given.

Subjects usually performed about ten runs in a session, each run lasting two or three minutes, and sessions lasting about half to one hour. Subjects were permitted to rest as they wished between runs.

2.5 Sequence and timing of events in an experiment

In all the experiments reported here, each stimulus or pair of stimuli was presented for 100 msec. This time is too short for voluntary changes in fixation during a presentation (Westheimer, 1954; Bartz, 1962; White & Eason, 1962). That is, once the stimuli had appeared, the subject could not make a voluntary saccade to a new position while the stimuli were still on display. This is crucial to the design of the experiments as it precludes (i) the involvement of eye-movements in the comparison of patterns, and (ii) deliberate fixation at the position of eccentric stimuli. This latter is precluded because in all experiments using eccentric stimuli either the subject had no *a priori* knowledge of where the stimulus would occur, or two stimuli of equal and opposite eccentricities would appear simultaneously. The importance of this latter point will become apparent later.

The typical timing sequence in a trial was this: following initiation of the trial by the subject, the fixation spot (or fixation pattern) was extinguished and, after a 1.0 sec delay the stimulus pair appeared for 100 msec (in sequential experiments, the first stimulus pattern appeared for 100 msec; after a further 1.0 sec delay the second stimulus pattern appeared for 100 msec). When the subject responded, the remainder of the fixation display was extinguished, the subject's response was recorded by the computer, and after a further 1.0 sec delay the fixation pattern and spot were redisplayed, indicating that the next trial could be started.

The timing of the fixation display varied among experiments. For experiments in which a fixation spot would not interfere with the stimulus, the spot was displayed throughout the presentation, and a fixation pattern such as a cross would appear only at the start of a trial. Otherwise a display of lines pointing to the point of fixation would be displayed throughout the presentation, and a fixation spot would appear only at the start of the trial.

2.6 Experimental design

In all the experiments and for each subject the sequence of experimental treatments was chosen randomly but balanced over runs for order and carry-over effects (Finney, 1960). The design varied among experiments, but it was always based on some variant of the following technique. The number of experimental treatments n is chosen so that n+1 is a prime number. Then for a given experimental subject, the 1^{th} experimental treatment is assigned a number in the sequence 1,2,..., n at random. This assignment is denoted T_1. [Note that although all the 'same' treatments were distinguishable from one another, this is not the case for the 'different' treatments. For example, one cannot sensibly define the angle through which one pattern has been rotated from a different pattern. This does not affect the balancing procedure.] Then if an experiment is performed in n blocks of n runs, the experimental treatment in the i^{th} trial of the j^{th} run is T_k where $k= (ij) \bmod (n+1)$. This design has the property that it is maximally balanced for order and carry-over effects. This holds true even in the case where a fixed treatment is assigned to more than one number in the sequence 1,2,..., n (as in the case of 'differents').

In all the experiments the number of 'different' treatments was equal to the number of 'same' treatments, to counteract the possibility of 'response-determined' responses (Senders & Sowards, 1952). It should be noted that subjects were not informed of the proportions of 'sames' and 'differents' in an experiment.

2.7 Data analysis techniques

In all the experiments except one, the data were analysed in terms of the discrimination index d' (Green & Swets, 1966). The advantage of using d' is that it gives a measure of discrimination performance which is, to first order, independent of the subject's response bias, and so allows comparison between experimental treatments and between subjects. This measure is used as a first order approximation rather than because of its psychophysical significance; all that is required is that d' is monotonic with discriminability. d' has the properties that it is zero when the stimuli under comparison are not discriminated, and increasing positive d' indicates increasing discriminability.

One can regard the experimental task as a discrimination task, equating the detection of 'same' pairs to the detection of a signal in noise, where the noise is made up of 'different' pairs. The d' model is as follows: the sensation arising from a noise-stimulus ('different' pair) is normally distributed in one dimension, with mean μ_n and standard deviation σ_n; the distribution of sensation arising from a signal ('same' pair) has mean μ_s and standard deviation σ_s. The subject reports that a signal is present if the sensation exceeds his criterion value c. Under the equal variance model, which we use here in the absence of more detailed knowledge appropriate to this type of experiment, $\sigma_s = \sigma_n = \sigma$. Experimentally, we have access to estimates of two parameters; the probability of the correct detection of a signal, p_d, and the probability of a signal report in the absence of a signal (false alarm), p_{FA}. Now under the model, if we adjust the axis scale so that $\sigma = 1$,

$$p_d = \frac{1}{\sqrt{2\pi}} \int_c^\infty e^{-\frac{(z-\mu_s)^2}{2}} dz = \frac{1}{\sqrt{2\pi}} \int_{z_1}^\infty e^{-\left(\frac{z}{2}\right)^2} dz$$

where $z_1 = c - \mu_s$; similarly

$$p_{FA} = \frac{1}{\sqrt{2\pi}} \int_{z_2}^\infty e^{-\left(\frac{z}{2}\right)^2} dz$$

where $z_2 = c - \mu_n$. Now $z_2 - z_1 = \mu_s - \mu_n$.

If we define $d' = \mu_s - \mu_n$, the distance between the two means, we find that we have a measure of discriminability which is criterion-independent. d' is in units of σ. Now we can use a table of the standard normal distribution (or an appropriate algorithm), to look up the values of z_1 and z_2 which correspond to the values p_d and p_{FA} respectively.

Variances for d' were calculated using the method of Gourevitch & Galanter (1967);

$$\sigma_{d'}^2 = \frac{2\pi p_d(1-p_d)}{n_{same}\, e^{-z_1^2}} + \frac{2\pi p_{FA}\,(1-p_{FA})}{n_{diff}\, e^{-z_2^2}} ,$$

where $\sigma_{d'}$ is the estimated standard deviation of d',

n_{same} is the number of 'same' trials

n_{diff} is the number of 'different' trials.

This estimate is based on a local linear approximation.

For statistical tests $z = d'/\sigma_{d'}$ can be treated as a standard normal variable. To pool data across a number of subjects $i = 1, 2, \ldots, n$, one can use

$$z_{av} = \frac{1}{n} \sum_{i=1}^{n} d'_i \quad / \quad [\frac{1}{n^2} \sum_{i=1} \sigma^2_{d'_i}]^{\frac{1}{2}}$$

as a standard normal variable. The statistical tests quoted in the text are trend analyses and contrast tests as described by Lindman (1974). In some cases chi-squared tests for inter-subject differences were carried out. In these tests, the discrimination indices d'_{ij} and their variances v_{ij} were calculated, where $i = 1, 2, \ldots, n_s$ specifies the subject and $j = 1, 2, \ldots, n_t$ specifies the experimental treatment. Under the hypothesis that there are no differences between subjects' performances, the quantity

$$\chi^2 = \sum_{ij} \quad (d'_{ij} - \bar{d}'_{.j})^2 \;/\; v_{ij}$$

where $\quad \bar{d}'_{.j} = \frac{1}{n_s} \quad \sum_i d'_{ij} \quad ,$

should be distributed as chi-squared with $n_t \; (n_s-1)$ degrees of freedom (Kendall and Stuart, 1977). To test for differences between subjects, allowing for each subject's overall performance level, the mean performance level for each subject $\bar{d}'_{i.} \;=\; \frac{1}{n} \; \sum_j \; d'_{ij}$ was subtracted from his d' scores to give $e_{ij}=d'_{ij}-\bar{d}'_{i.}$. Under the hypothesis that there are no differences between subjects' performances when each of these is expressed relative to the subject's mean performance level, the quantity

$$\chi^2 = \sum_{ij} \; (e_{ij} - \bar{e}_{.j})^2 \quad / \quad v_{ij}$$

should be distributed as chi-squared with $(n_t-1)(n_s-1)-1$ degrees of of freedom.

In most of the experiments it was assumed that there were no differences between subjects apart from their mean levels of performance. The chi-squared tests mentioned above disconfirmed this hypothesis for only one experiment, and therefore for the rest of the experiments, d' and its variance was pooled across subjects to simplify the analysis. Where the hypothesis was disconfirmed, the data were again pooled, but variances were estimated using the standard error in the means of the d's rather than the pooled variance mentioned above. In this case, the data can only be regarded as representing underlying means in the population of subjects; the data do not reflect the way an individual subject can be expected to perform.

3. THE EFFECTS OF POSITIONAL SYMMETRY AND SEPARATION ON THE VISUAL COMPARISON OF TRANSFORMED PATTERNS

From the evidence discussed in Section 1.3 it is clear that the arrangement of stimuli in the visual field has important effects on the way in which the stimuli are perceived, although the precise nature of these effects is not clear. The experiments reported in this Chapter are an attempt to clarify the effects of two parameters of the stimulus arrangement. For a pair of stimulus patterns, these parameters are:

(a) the symmetry of the positions of the patterns with respect to the point of fixation;

(b) the separation of the positions of the patterns.

The parameters (a) and (b) were chosen in the light of the results of some pilot experiments. These experiments concerned the phenomenon that when subjects make 'same-different' comparisons of pairs of simultaneously presented patterns, one of which is a version of the other that has been rotated in the plane, recognition performance is low in the region of 90° rotation angle and subsequently rises in the region of 180° (see Section 1.2.2). This elevation of performance at 180° relative to 90° was found to be reduced or even abolished if patterns were presented either sequentially in the same central position or simultaneously in an asymmetric arrangement of positions. For this reason a systematic study of the effects of positional symmetry and separation was performed.

It should be emphasized that the parameter of symmetry is meant to refer to the symmetry of the *positions* of the stimuli with respect to the point of fixation, and not to the symmetry of the stimuli themselves.

Four experiments concerning this topic were performed. Experiment 3.1 is an investigation of the effects of (a) and (b) on the 'same-different' comparison of stimulus pairs which were related by one of four possible transformations. These were identity, planar rotation through 90°, planar rotation through 180° (point inversion), and reflection about a vertical axis.

Experiment 3.2 demonstrates that the effects found in Experiment 3.1 do not depend on whether the presentation of the two patterns in a pair is sequential or simultaneous.

Experiment 3.3 demonstrates that the effects found in Experiment 3.1 are not specific to the stimuli used in that experiment.

Experiment 3.4 is an investigation of the effects of (a) and (b) in finer detail, for two of the transformations.

In all these experiments the subject's task was to decide if the two patterns in a pair were 'same', taking into account possible rotations or reflections, or 'different'.

3.1 Experiment 3.1.

The purpose of this experiment was to systematically investigate the effects of positional symmetry and separation on the 'same-different' comparison of transformed patterns. The systematic variation in these parameters was achieved by positioning each of the stimuli in one of three positions. These were:

(i) 0.5° to the left of the fixation point;

(ii) on the fixation point;

(iii) 0.5° to the right of the fixation point.

The use of all possible pairs of these positions made it possible to measure the following effects:

(a) the effects of positional symmetry with separation held constant;

(b) the effects of separation with positional symmetry held constant;

(c) the effects of varying both positional symmetry and separation at once.

3.1.1 Methods

Subjects. The subjects were five male students in the Department of Communication & Neuroscience, aged between 23 and 27 years. All had normal or corrected-to-normal vision, and all, except the author, were unaware of the purpose of the experiment.

The display. The display was as described in Section 2.1. Fixation was aided by four computer generated white lines, 0.5° long, pointing to a fixation spot. The lines were displayed throughout each presentation; the spot was extinguished at the start of each trial.

Stimuli. The stimuli were random dot patterns as described in Section 2.3.

Pattern positions. In each trial two patterns appeared sequentially. Each pattern was presented in one of three positions:

(a) eccentric, with the pattern centre 0.5° to the left of the fixation spot;

(b) centred on the fixation spot;

(c) eccentric, with the pattern centre 0.5° to the right of the fixation spot.

All possible pairs of these positions were used; the pairs were classified into four groups ('position combinations') in terms of the positional symmetry and separation of the patterns (see Fig. 3.1 for a schematic illustration of this):

E_s: both patterns eccentric on the same side (left or right). Here the separation is zero and the patterns are symmetrically arranged.

E_c: one pattern eccentric (left or right), the other central. Here the separation is 0.5° and the arrangement is closer to symmetry.

E_o: both patterns eccentric and on opposite sides. Here the separation is 1.0° and the arrangement is symmetrical.

C_c: both patterns central. Here the separation is zero and the arrangement is symmetric.

It will be noted that there are two 'sub-pairs' in E_s (left-left and right-right); four 'sub-pairs' in E_c (left-centre, centre-left, right-centre, and centre-right); two 'sub-pairs' in E_o (left-right and right-left); and one pair in C_c. For the purpose of analysis, all the 'sub-pairs' of a given position combination were treated as equivalent.

Pattern transformations. There were four possible transformations relating the patterns in a 'same' pair. [Strictly, the translations determined by the positions of the patterns should also be considered as transformations but, for the reasons outlined at the beginning of this Chapter, position is treated separately from the other transformations.] The transformations were:

Id: the two patterns were identical;

Ro: one pattern was obtained from the other by planar rotation through 90° (either clockwise or anti-clockwise);

Pi: one pattern was obtained from the other by point inversion, that is, planar rotation through 180°;

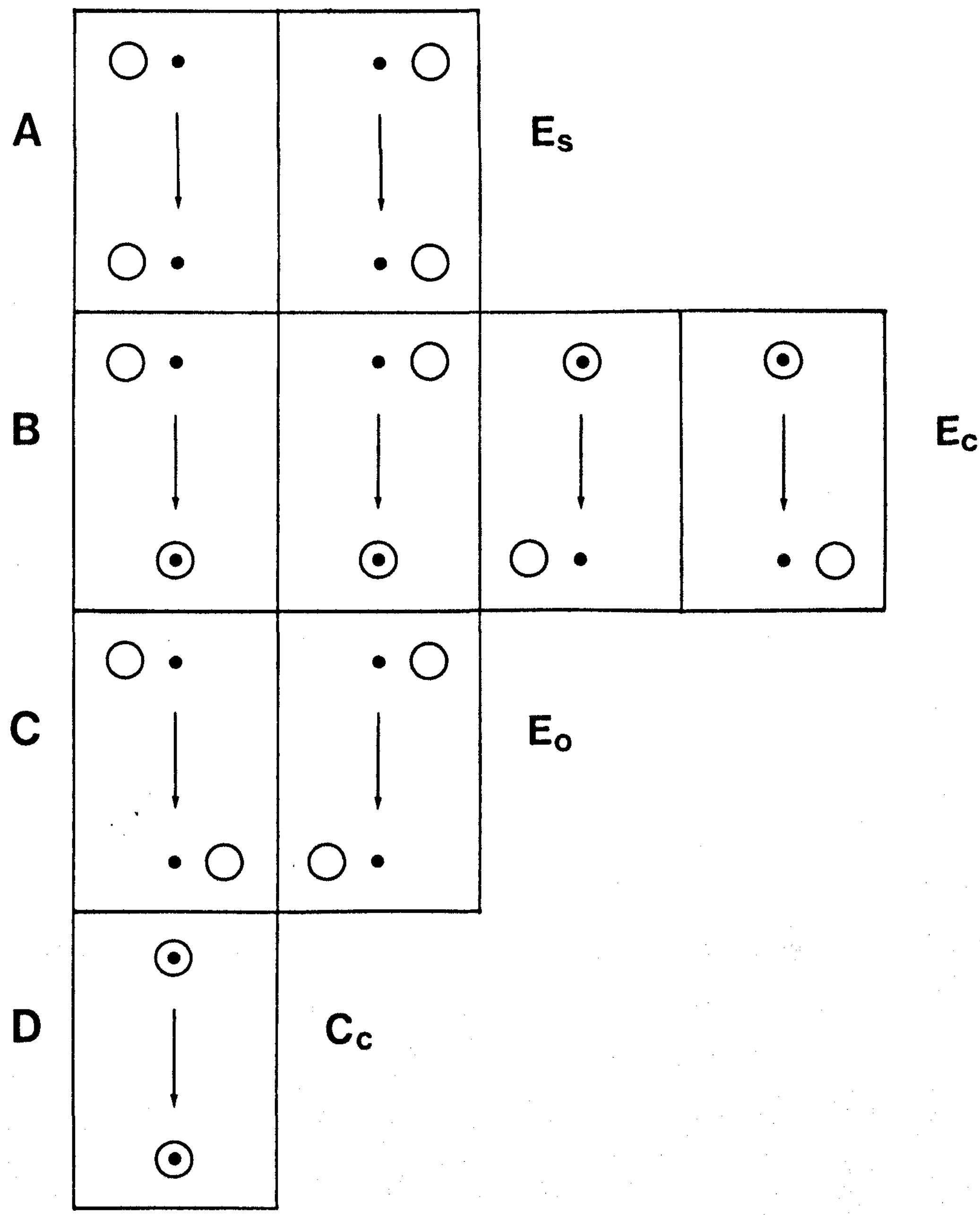


Fig 3.1

Fig. 3.1. Schematic diagram of the 'position combinations' (Es, Ec, Eo, Cc) in Experiment 3.1. The circles represent dot patterns, the black dots represent the fixation point, and the arrows represent the 1.0 sec delay between the two patterns in a trial.

Mi: one pattern was obtained from the other by reflection in a vertical line.

For 'different' pairs, two independent patterns were generated.

A fresh pattern or pair of patterns was generated for every trial.

Instructions and presentation sequence. The instructions to the subject and the sequence of events in a presentation were as described in Sections 2.4 and 2.5. The subject's task was to decide if the two patterns in a pair were 'same', taking into account possible rotations or reflections, or 'different'.

Experimental design. In each run every position (E_s, E_c, E_o, and C_c) occurred once with each of the four 'same' pattern transformations (Id, Ro, Pi and Mi) and four times with 'differents', so that a run consisted of 16 'sames' and 16 'differents'. Each subject performed 48 runs over a period of several days.

The number of treatments used in this experiment made the design complex. For the purpose of balancing the order of the pattern transformations, each run was divided into two sections of sixteen trials, since, for the balancing, the number of treatments in a run (16 here) must be one less than a prime number (see Section 2.6). Within each section, the order of the pattern transformations was chosen randomly, but balanced for order and carry-over effects over runs (see Section 2.6). Thus in each section there were eight 'different' pairs and two each of the 'same' transformation pairs. For each subject, two new random sequences were generated before each set of sixteen runs, and these were permuted each run to implement the balancing.

The sequence of position combinations occurring with a given pattern transformation was chosen randomly but balanced for order and carry over effects over runs. A new random sequence for this purpose was generated every four runs and permuted each run.

The order of occurrences of the 'subpairs' of each position combination was chosen randomly so that, in four runs, each 'subpair' occurred the same number of times with every pairing of position combination and pattern transformation. This had the effect that the subject had no *a priori* knowledge of the position in which any of the patterns would appear.

Finally, the Ro transformation occurred the same number of times as a clockwise or anticlockwise rotation in every run.

3.1.2 Results

Fig. 3.2 shows 'same-different' pattern discrimination performance in Experiment 3.1. In each graph the discrimination index d' (see Section 2.7) is plotted against pattern transformation. Graphs A,B,C and D correspond to position combinations E_s, E_c, E_o, C_c.

The d' data are pooled over all the subjects. Chi-squared tests (see Section 2.7) on individual data revealed (i) significant differences between subjects' absolute performance levels ($p < 0.01$) and (ii) no significant differences between subjects, after allowing for each subject's overall performance level ($p > 0.5$).

Various tests for the effects of symmetry and separation were performed on the data. The significance levels quoted are the results of contrast tests and trend analyses (see Section 2.7).

(i) *Effect of distance with symmetry held constant.* The separation of the patterns is zero in position combination C_c (Fig. 3.2, D) and 1.0° in combination E_o (Fig. 3.2, C). In both cases the

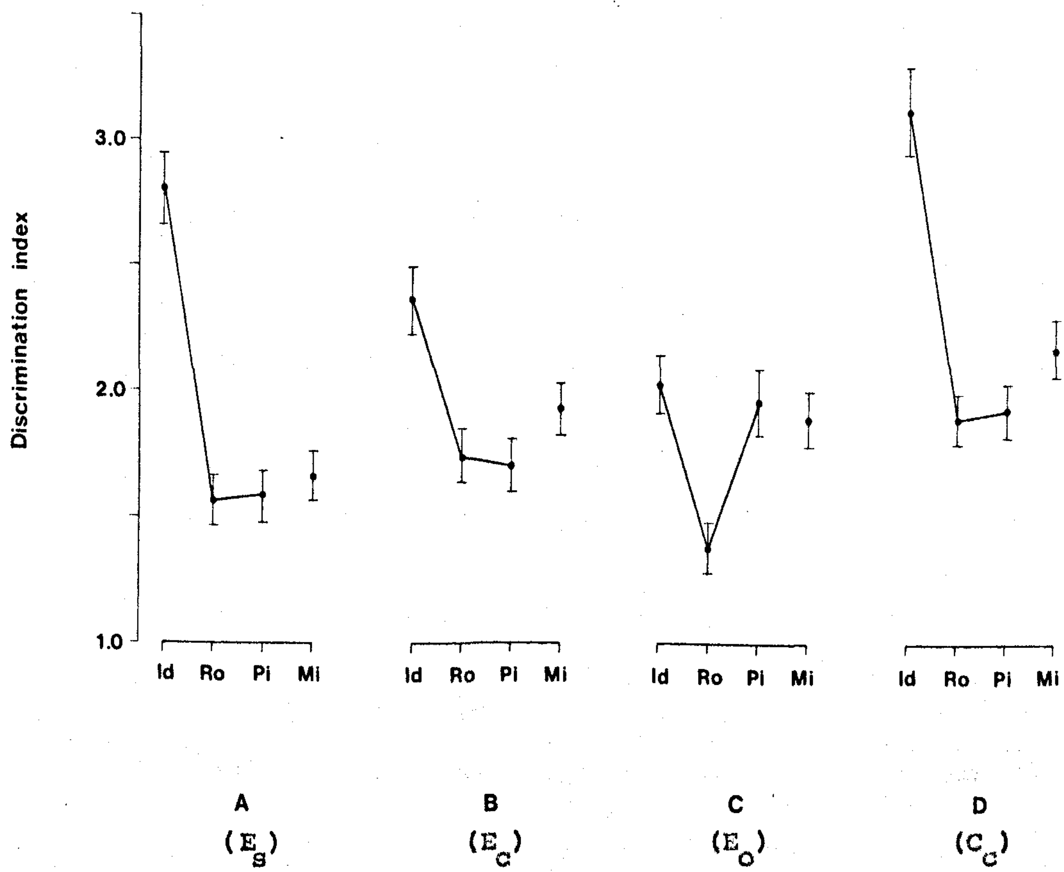


Fig 3.2

Figure 3.2. 'Same' detection performance in Experiment 3.1. Each graph shows the pooled discrimination index d' plotted against pattern transformation for one of the combinations of pattern positions. The position combinations are: A both patterns presented 0.5° to one side of fixation spot (combination Es); B one pattern presented 0.5° to the left or right of fixation spot, the other central (combination Ec); C one pattern presented 0.5° to the left of fixation spot, the other 0.5° to the right (combination Eo); D both patterns presented centrally (combination Cc). The pattern transformations are as follows. Id: the patterns are identical; Ro: the patterns are related by a 90° planar rotation; PI: the patterns are related by point-inversion; MI:

patterns are positioned symmetrically with respect to the point of fixation. The increase in separation of the patterns causes a large reduction in "same" detection performance for transformation Id ($p < 0.001$, 2-tailed test) and a significant reduction for transformation Ro ($p < 0.001$, 2-tailed test). There is no significant change in performance for Pi and a small but not significant change for Mi ($p > 0.5$, $p > 0.05$, respectively, 2-tailed tests).

(ii) *Effect of symmetry with distance held constant.* The patterns are positioned asymmetrically with respect to the point of fixation in the E_s position combination (Fig. 3.2, A) and positioned symmetrically in the C_c combination (Fig. 3.2, D). The patterns are not separated in either case. Introduction of symmetry while holding distance constant causes a small but not significant increase in "same" detection performance for transformation Id ($p > 0.1$, 2-tailed test) and significant increases in performance for transformations Ro, Pi, and Mi (respectively, $p < 0.05$, $p < 0.05$, $p < 0.001$, 2-tailed tests).

It is made clear below that both symmetry and distance effects cannot be ascribed to variations in acuity with retinal eccentricity.

(iii) *Combined effects of symmetry and distance.* The distance between the pattern positions increases linearly from the E_s position combination (Fig. 3.2, A) to the E_o combination (Fig. 3.2, C); the E_s combination is positionally asymmetric whereas the E_o combination is positionally symmetric. The E_c position combination (Fig. 3.2, B) is intermediate in both symmetry and distance. From E_s to E_o there is a linear decrease in "same" detection performance for transformation Id and a linear increase in detection performance for transformation Pi. [Linear trend in d' for Id is significant, $p < 0.001$, 2-tailed test, quadratic trend is not significant, $p > 0.05$, 2-tailed test. Linear trend in d' for Pi is significant, $p < 0.05$, 2-tailed test, quadratic trend is not significant, $p > 0.05$, 2-tailed test.] "Same" detection performance for transformation Mi shows no significant increase from E_s to E_o. (Linear and quadratic trends in d' are not significant, $p > 0.1$ and $p > 0.2$ respectively, 2-tailed tests.) "Same" detection performance for transformation Ro shows a non-linear trend from E_s to E_o (d' has had no significant linear trend and a significant quadratic trend, $p > 0.1$, $p < 0.05$ respectively, 2-tailed tests.)

Note that the marked qualitative differences between performance in combinations E_s and E_o cannot be ascribed to retinal-eccentricity and hence acuity effects: eccentricity is identical in the two cases.

To summarize, "same" detection performance for transformation Id is strongly affected by the distance between the patterns. Performance for transformations Pi and Mi is best when the patterns are positioned symmetrically, and the separation of the patterns then has no effect on performance for transformation Pi and a small effect on performance for transformation Mi. Performance for transformation Ro shows no simple dependence on either symmetry or separation. Performance is highest for the C_c position combination (Fig. 3.2, D), less for the E_c combination (Fig. 3.2, B) and lowest for the E_s and E_o combinations (Fig. 3.2, A and C). This suggests that it is more appropriate to consider performance for Ro as being determined by the mean distance of the patterns from the fixation point. In fact, d' for Ro shows a highly significant linear dependence on the mean distance ($p < 0.001$, 2-tailed test).

It might be suggested that the results of this experiment were an artifact of the sequential presentation. Although there was insufficient time during the presentation of each pattern for subjects to make directed shifts in the point of fixation based on the stimulus arrangement (see Section 2.4), subjects were able to do this between the presentations (although they were instructed not to do so). Eye-movements cannot, however, account for the results for two reasons. First, subjects had no a priori knowledge of the position in which any pattern was going to appear, so that any systematic strategy of eye-movements was balanced by the design of the experiment. For example, in position combination E_c, left-centre, centre-left, right-centre and centre-right occurred equally often and in pseudo-random order. Second, eye-movements cannot simply account for the fact that, as the positional symmetry and separation of the patterns increases, detection performance for transformation Id goes down, whereas detection performance for transformation Pi goes up. A systematic strategy of eye-movements might give an advantage to one particular position combination, but would do so independent of pattern transformation. Assume, for example, that after the first pattern has been presented, the subjects always fixated the position of that pattern, ready for the second pattern. Then performance would be high for position combinations C_c and E_s relative to E_c and E_o for all the pattern transformations. The results show that although this might be true of transformation Id, it is not true for the other transformations. The same arguments apply for any other systematic strategy that one can postulate; there is no simple eye-movement account of the results.

Another hypothesis, which cannot be so easily rejected, is that the results depend on the time difference between the patterns in a trial; that is, the results are a peculiarity of the temporally asymmetric memory matching that was required.

To demonstrate that the results do not depend on the sequential presentation, an experiment using simultaneous presentation was performed.

3.2 Experiment 3.2

In Experiment 3.2 pairs of patterns were presented simultaneously, side-by-side. The experiment was similar to Experiment 3.1, but since two patterns cannot be simultaneously presented in the same field position, the replication was limited to the E_o and E_c position combinations.

3.2.1 Methods

The subjects and methods for this experiment were the same as those for Experiment 3.1 with the following exceptions.

(i) The presentation of the two patterns in a trial was simultaneous.

(ii) The distance of the eccentric positions from the fixation point was increased to 1.0°, so that in the E_c position combination the patterns would be well separated.

(iii) The experiment differed in design. In each run, each position combination occurred twice with every 'same' pattern transformation, and eight times with 'differents', so that a run consisted of sixteen 'sames' and sixteen 'differents'. Each subject performed 32 runs over a period of several days. For the purpose of balancing, each run was split into two sections of sixteen trials. Within each section, the order of the pattern transformations and position combinations was chosen randomly but balanced for order and carry-over effects over runs (see Section 2.6). For each subject two new random sequences were generated for every sixteen runs, and were permuted every run to implement the balanced design. The E_c position combination occurred once as centre-left and once as centre-right with every pattern transformation in each run.

3.2.2 Results

Fig. 3.3 shows 'same-different' pattern discrimination performance in Experiment 3.2. The pooled discrimination index d' (see Section 2.7) is plotted against pattern transformation for the position combinations E_c (Fig. 3.3, A) and E_o (Fig. 3.3, B). The d' data are pooled over subjects. Chi-squared tests (see Section 2.7) on individual data revealed: (i) significant differences between subjects' absolute performance levels ($p < 0.001$); (ii) no significant differences between subjects after allowing for each subject's absolute performance level ($p > 0.2$).

The statistical tests reported below are contrast tests (see Section 2.6). In the E_c combination (Fig. 3.3, A) the pattern positions are asymmetric with respect to the point of fixation and the distance between the patterns is 1.0°. In the E_o position combination (Fig. 3.3, B) the pattern separation is 2.0° and the positions are symmetric with respect to the point of fixation. The increase in distance and symmetry going from E_c to E_o reduces "same" detection performance for transformation Id ($p < 0.001$, 1-tailed test) and increases performance for transformation Pi ($p < 0.01$, 1-tailed test). There is no increase in performance for transformation Mi or reduction for transformation Ro ($p > 0.2$ for both, 1-tailed tests).

It thus appears that the results of Experiment 3.1 are not a consequence of the difference in the times of presentations of patterns in a trial, or an artifact of eye-movements between the two presentations.

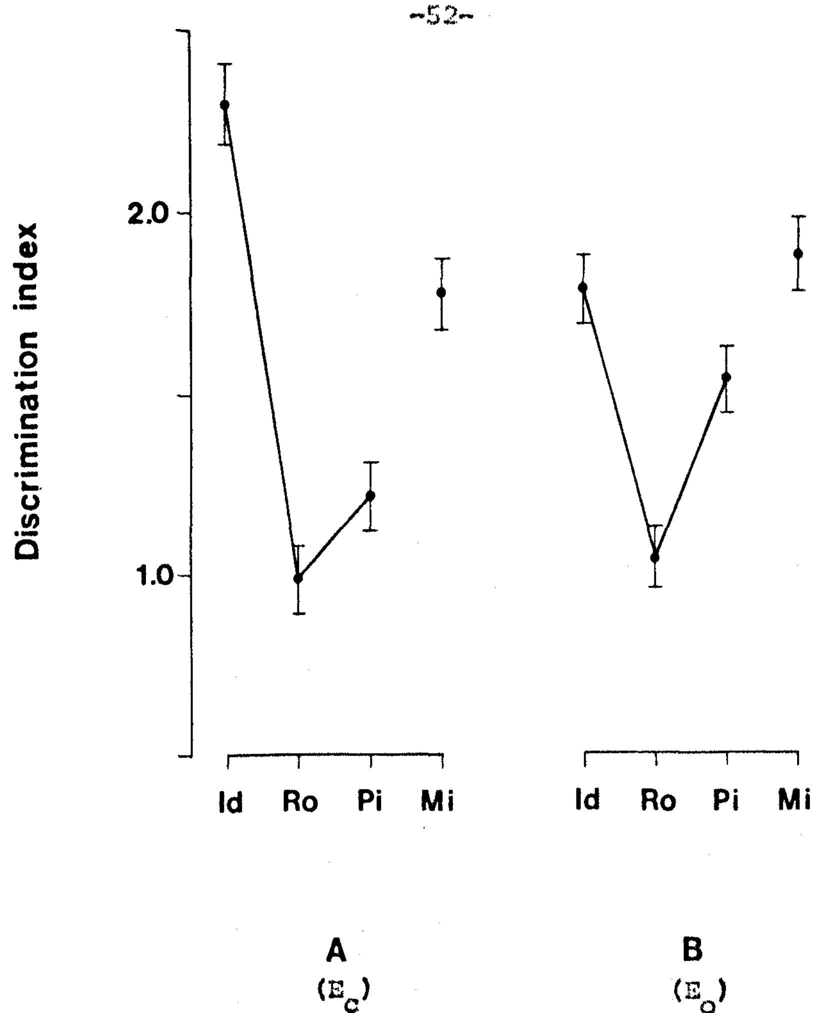


Fig 3.3

Figure 3.3. 'Same' detection performance in Experiment 3.2. The pooled discrimination index d' is plotted against pattern transformation, values Id, Ro, Pi and Mi, for the two combinations of pattern position: A one pattern presented 1° to the left or right of fixation spot, the other central (combination Ec); B one pattern presented 1° to the left of fixation spot, the other 1° to the right (combination Eo).

3.3 Experiment 3.3

To give some generality to the results of Experiment 3.1, the experiment was repeated with a different type of stimulus. Since it was desired to retain the unfamiliarity property of random dot patterns (see Section 2.3), random line patterns were chosen as the new stimuli.

3.3.1 Methods

Apart from the nature of the stimuli, all the methods of this experiment were identical to those of Experiment 3.1. The subjects were the author and one other subject who took part in Experiment 3.1.

The random line patterns consisted of ten white lines generated within an imaginary disc of 0.5° diameter. The end points of the lines were constrained to be at least 0.025° apart; apart from the above restrictions, the lengths of the lines were randomly distributed between 0.15° and 2.0°, and their orientations were also random. Some examples of the line patterns are shown in Fig. 3.4, and a flowchart of the pattern generating program is shown in Fig. 3.5.

3.3.2 Results

Fig. 3.6 shows 'same-different' pattern discrimination performance in Experiment 3.3. In each graph the discrimination index d' (see Section 2.7) is plotted against pattern transformation. Graphs A,B,C, and D correspond respectively to the position combinations E_s, E_c, E_o,

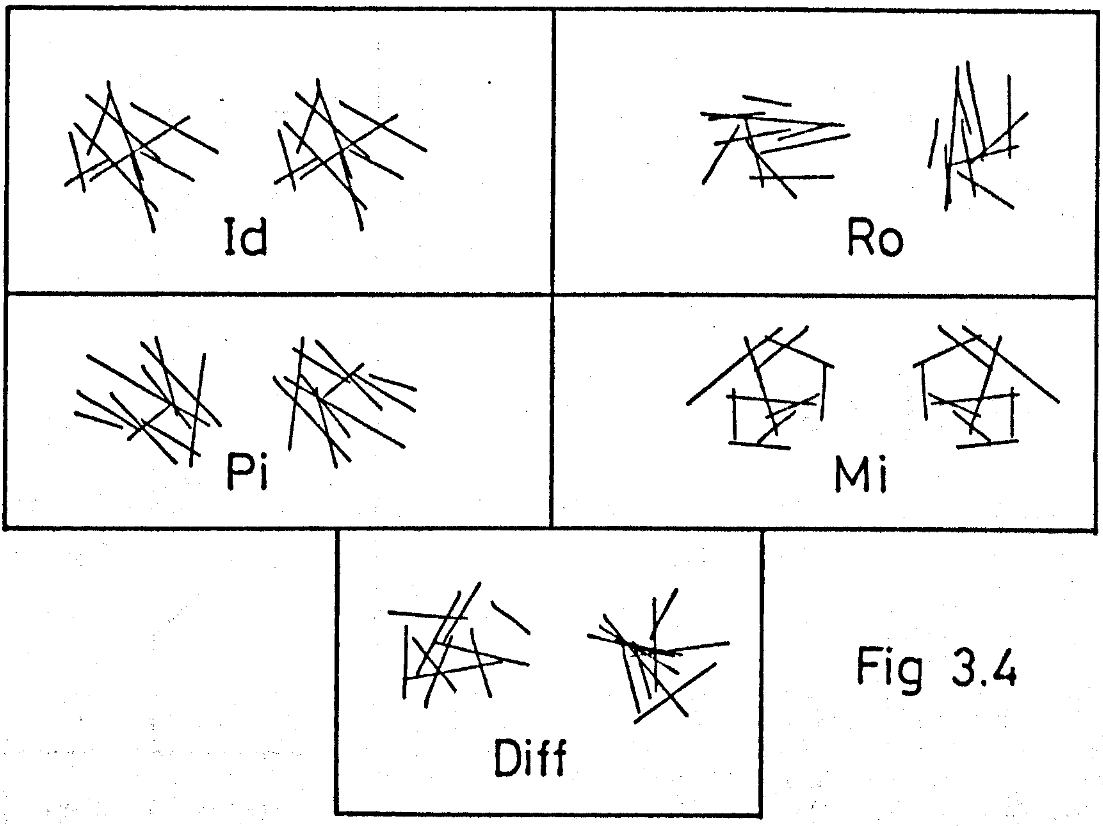


Fig 3.4

Figure 3.4. Random line patterns used as stimuli in Experiment 3.3.

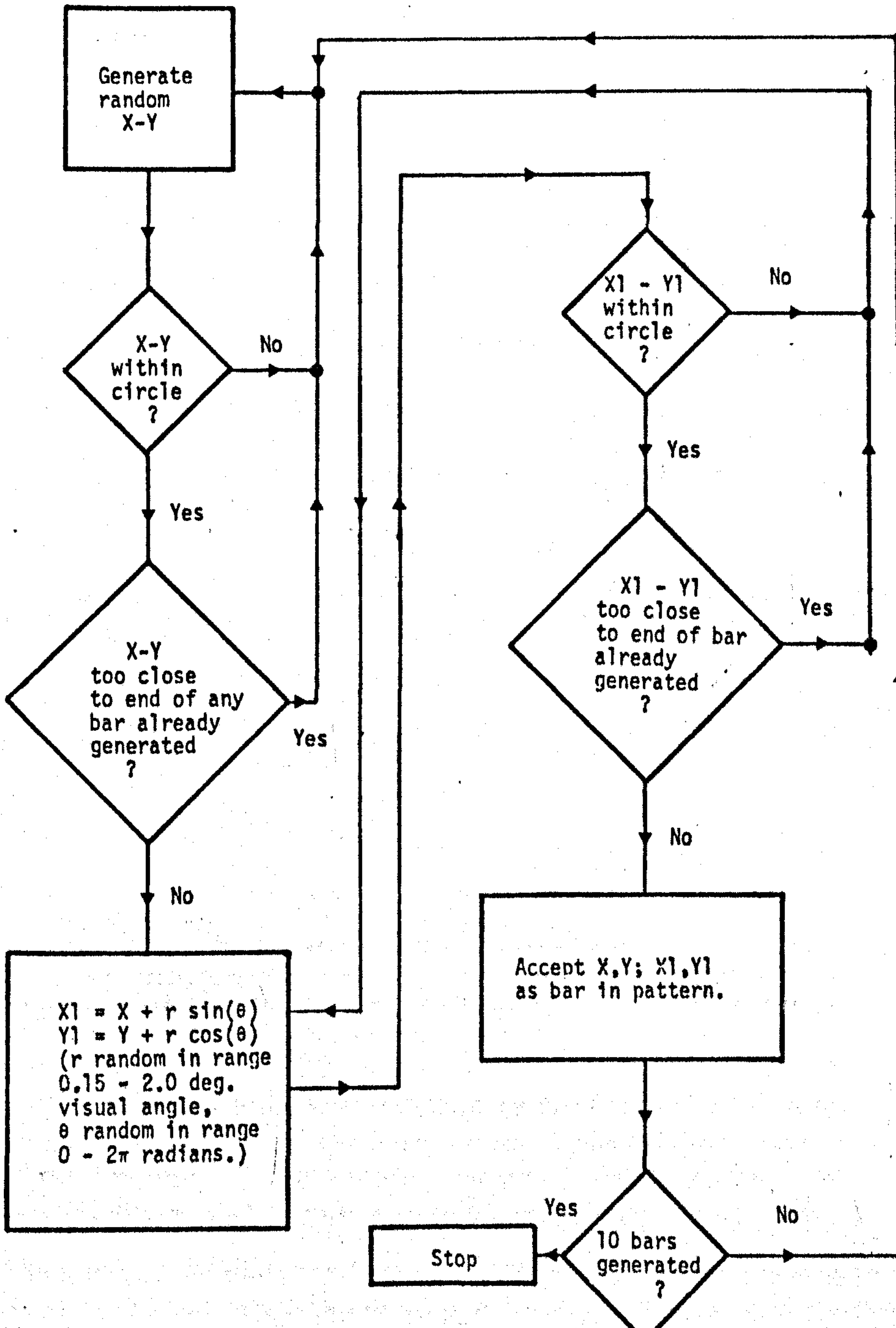


Fig 3.5

Figure 3.5. Flowchart for the generation of the random line patterns used as stimuli in Experiment 3.3.

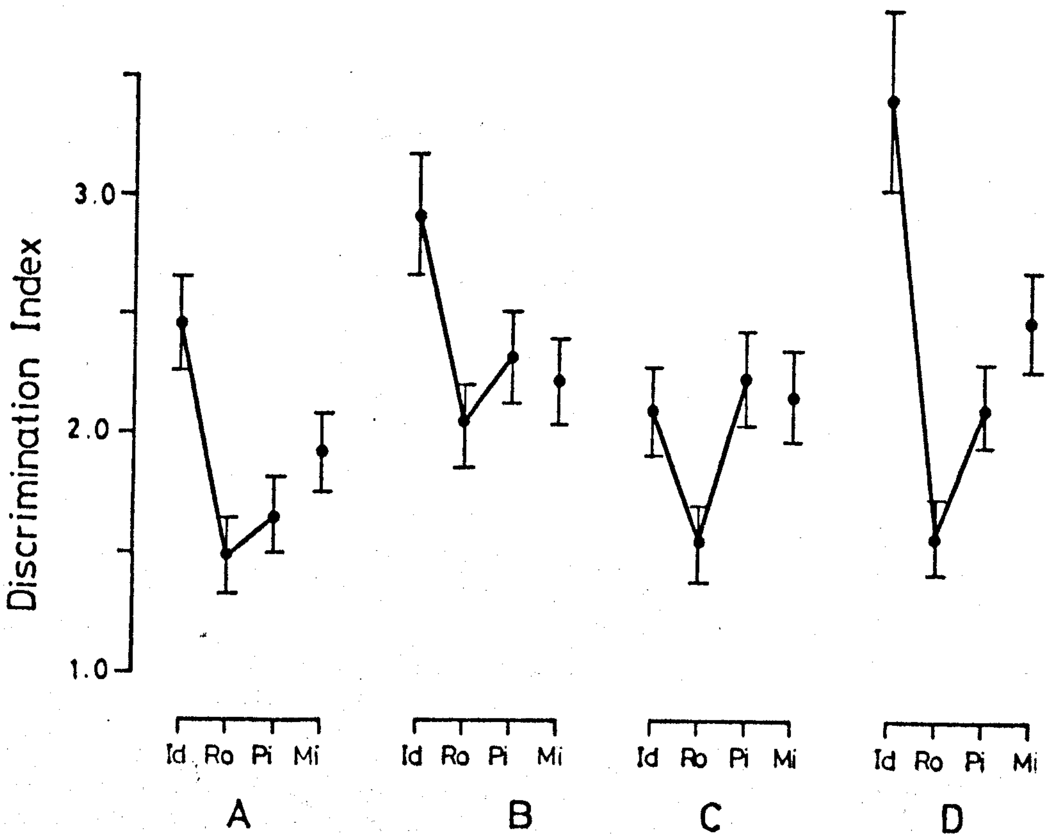


Fig 3.6

Figure 3.6. 'Same' detection performance in Experiment 3.3. Each graph shows the pooled discrimination index d' plotted against pattern transformation for one of the combinations of pattern positions. The position combinations are: A both patterns presented 0.5° to one side of fixation spot (combination Es); B one pattern presented 0.5° to the left or right of fixation spot, the other central (combination Ec); C one pattern presented 0.5° to the left of fixation spot, the other 0.5° to the right (combination Eo); D both patterns presented centrally (combination Cc). The pattern transformations are as follows. Id: the patterns are identical; Ro: the patterns are related by a 90° planar rotation; Pi: the patterns are related by point-inversion; Mi: the patterns are related by reflection in a vertical line.

and C_c.

The d' data are averaged over the two subjects. Chi-squared tests (see Section 2.7) on the individual data revealed no significant differences between the two subjects' absolute performance levels ($p > 0.4$), and no significant differences between the subjects' absolute performance levels in this experiment and those in Experiment 3.1 ($p > 0.5$).

The same comparisons were made on the data as were made on the data of Experiment 3.1, so only the results of the tests will be given here (see Section 3.1.2 for details).

(i) Effect of distance with symmetry held constant. There is a significant effect of distance on performance for transformation Id ($p < 0.01$, 2-tailed test) and there are no significant effects of distance for transformations Ro, Pi or Mi ($p > 0.8$, $p > 0.6$, $p > 0.2$, respectively, 2-tailed tests).

(ii) Effect of symmetry with distance held constant. Unlike in Experiment 3.1, there is a significant effect of symmetry on performance for transformation Id ($p < 0.05$, 2-tailed test), and no significant effect for transformation Ro ($p > 0.6$, 2-tailed test). There is a significant effect of symmetry for both transformations Pi and Mi ($p < 0.05$, in both cases, 2-tailed tests).

It is made clear below that both symmetry and distance effects cannot be ascribed to variations in acuity with retinal eccentricity.

(iii) Combined effect of symmetry and distance. In the comparison along E_s, E_c, E_o there is a small but not significant linear trend in performance for transformation Id ($p > 0.05$, 2-tailed test) and a significant quadratic trend ($p < 0.05$, 2-tailed test). There is no significant linear trend in performance for transformation Ro ($p > 0.8$, 2-tailed test) and a significant quadratic trend ($p < 0.05$, 2-tailed test). There is a significant linear trend in performance for transformation Pi ($p < 0.05$, 2-tailed test) and no significant quadratic trend ($p > 0.05$, 2-tailed test). Neither linear nor quadratic trends in performance for transformation Mi are significant ($p > 0.2$, $p > 0.4$, respectively, 2-tailed tests).

Note that, as before, the marked qualitative differences between performance in combinations E_s and E_o cannot be ascribed to retinal-eccentricity and hence acuity effects: eccentricity is identical in the two cases.

Although these results are not identical to those of Experiment 3.1, there are strong similarities between the two sets of results. Performance for transformation Pi and Mi is significantly affected by positional symmetry (compare A with D, Fig. 3.6) and is not significantly affected by the distance between the patterns (compare C with D, Fig. 3.6). Performance for transformation Id is affected by both symmetry and distance, but the effect of distance is larger (compare A with C, Fig. 3.6). Given that the chi-squared test mentioned above showed no significant differences between performance in the two experiments, it is reasonable to conclude that the effects of positional symmetry and separation are not grossly changed by the use of a different stimulus.

3.4 Experiment 3.4

The final experiment in this Chapter was designed to investigate the effects of positional symmetry and separation on performance for transformations Id and Pi in finer detail. The experiment was also related to an experiment reported in Chapter 7, which has the same design, using different transformations. In each trial, as in Experiment 3.1, two stimuli were presented sequentially, each stimulus appearing in one of a number of possible positions. In this experiment, there were five positions; relative to the fixation point these were 1.0°, 0.5°, 0.0° to the left, and 0.5° and 1.0° to the right. By using all (non-equivalent) pairs of these it was possible to measure the effects of positional symmetry and separation in a 2-dimensional trend analysis.

3.4.1 Methods

Subjects. The subjects were three male students in the Department of Communication and Neuroscience, and one female visitor to the Department. All had normal or corrected-to-normal vision and all, except the author, were unaware of the purpose of the experiment.

The display. The display was as described in Section 2.1. Fixation was aided by two computer generated white lines, about 0.9° long, about 0.6° above and 0.6° below and pointing to a fixation spot. The lines were displayed throughout each presentation; the spot was extinguished at the start of each trial.

Stimuli. The stimuli were random dot patterns as described in Section 2.3.

Pattern positions. In each trial two patterns appeared sequentially. Each pattern was presented with its centre in one of five positions; 1.0°, 0.5°, 0.0° to the left of the fixation spot, and 0.5° and 1.0° to the right of the fixation spot. These positions will be referred to as a,b,c,d and e respectively.

The combinations of these positions were as follows:

Combination	Separation (units 0.5°)	Asymmetry (units 0.5°)
aa	0	4
ab	1	3
ac	2	2
ad	3	1
ae	4	0
bb	0	2
bc	1	1
bd	2	0
cc	0	0

Asymmetry is measured as the distance from one position to the reflection of the other in the fixation point.

For the purpose of analysis, the position combinations which are mirror equivalents are not distinguished. Similarly, the sequence of the positions is not taken into account in the analysis. For example, in the analysis, ab refers to the following four 'sub-pairs':

1st pattern	2nd pattern
1° left	0.5° left
1° right	0.5° right
0.5° left	1° left
0.5° right	1° right

All possible 'subpairs' occurred equally often with each of the pattern transformations, as will be indicated below.

Pattern transformations. There were two possible pattern transformations relating the patterns in each 'same' pair:

Id: the two patterns were identical;

Pi: one pattern was obtained from the other by point inversion, that is, planar rotation through 180°.

For 'different' pairs, two independent patterns were generated.

A fresh pattern or pair of patterns was generated for every trial.

Instructions and presentation sequence. The instructions and the sequence of events in a presentation were as described in Sections 2.4 and 2.5.

Experimental design. In each run, every position combination (ab, ac etc.) occurred once with each of the 'same' pattern transformations (Id and Pi), and twice with 'differents', so that a run consisted of 18 'sames' and 18 'differents'. Each subject performed 36 runs over several days.

The order of the pattern transformations and position combinations was chosen randomly but balanced for order and carry-over effects over runs (see Section 2.6). For each subject, a random sequence was generated before the experiment, and was permuted each run to implement the balanced design. The order of the 'subpairs' (mentioned above) of each position combination was chosen randomly so that, in four runs, each subpair occurred the same number of times with every pairing of pattern transformation and position combination. This had the effect that the subject had no *a priori* knowledge of the position in which any pattern would appear.

3.4.2 Results

Fig. 3.7 shows 'same-different' pattern discrimination performance in Experiment 3.4. In each graph, the discrimination index d' (see Section 2.7) is plotted against position combination.

The upper graph shows performance for transformation Id, and the lower graph shows performance for transformation Pi.

The d' data are pooled over subjects. Chi-squared tests on individual data revealed (i) significant differences between subjects' absolute performance levels ($p < 0.001$) and (ii) no significant differences between subjects, allowing for each subject's overall performance level ($p > 0.1$).

In the graphs of Fig. 3.7, the degree of positional asymmetry increases along position combinations cc, bb, aa, and for those points, the separation of the pattern positions is zero. The separation of the patterns increases along position combinations cc, bd, ae and, for those points, the positions of the patterns

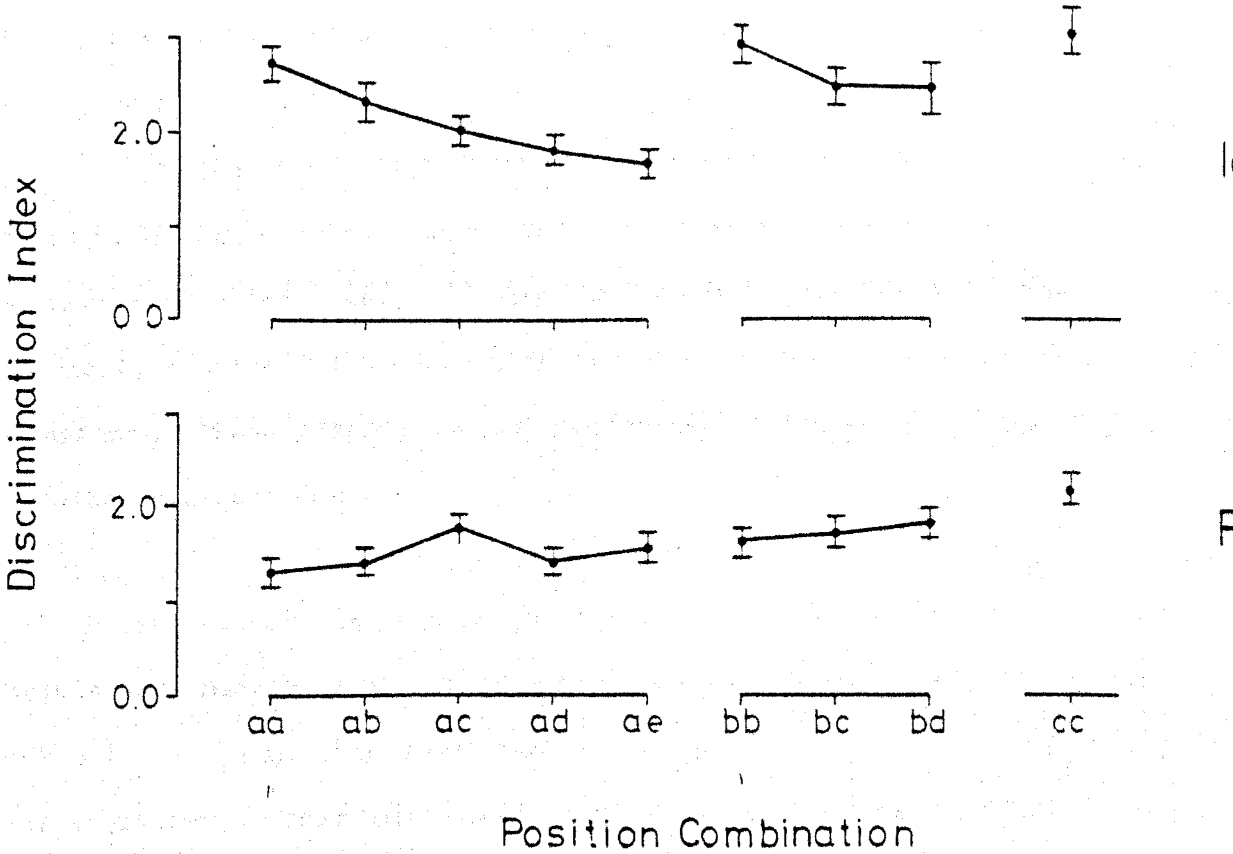


Fig 3.7

Figure 3.7. 'Same' detection performance in Experiment 3.4. Each graph shows the pooled discrimination index d' plotted against position combination.

are symmetric with respect to the point of fixation. Intermediate points are intermediate in positional symmetry and separation.

The upper graph (transformation Id), performance is nearly constant with asymmetry, and falls off considerably with separation; on the lower graph (transformation Pi), performance is nearly constant with separation, and falls off considerably with asymmetry. These effects are not unexpected in the light of the results of Experiment 3.1.

A statistical analysis was performed using contrast tests for trends (see Section 2.7). Two special tests were developed: (i) a test for a linear trend with separation in 2-dimensions; (ii) a similar test for asymmetry. These were based on the following theory (Lindman, 1974). In linear regression theory, the best estimate for a and b in the equation

$$y = ax+b$$

is

$$a = \frac{\sum_i (x_i - \bar{x})y_i}{\sum x_i^2 - n\bar{x}^2}$$

where y is the dependent variable, and y_i is the observed value of y at the i^{th} point x_i.

To test for a linear trend, for example, one tests the null hypothesis H_0: $a = 0$ against the competing hypothesis $H_1 : a \neq 0$. This is equivalent to testing

$$H_0 : \sum_i (x_i - \bar{x})y_i = 0$$

against

$$H_1 : \sum_i (x_i - \bar{x})y_i \neq 0$$

This constitutes a linear contrast with coefficients

$$c_i = x_i - \bar{x}$$

or, more conveniently, $c_i = nx_i - \sum x_i$

where n is the number of points x_i. Now if each y_i is normally distributed with variance v_i, under the null hypothesis

$$c = \sum c_i y_i \; / \; (\sum c_i^2 v_i)^{\frac{1}{2}}$$

is a standard normal variable.

The variables x_i are *separation* in test (i) and *asymmetry* in test (ii). (Asymmetry is measured as the distance of the position of one pattern to the reflection of the position of the other in the fixation point.) These values and the coefficients derived from them are displayed in Table 3.1.

All the points of Fig. 3.7 are included in the tests, with the d' values as the y_i and their variances as the v_i. Separate tests were carried out on performance for transformations Id and Pi.

(i) The effect of separation. The tests for the effects of separation revealed (a) a highly significant effect of separation on performance for transformation Id ($p < 0.001$, 2-tailed test) and (b) no significant effect of separation on performance for transformation Pi ($p > 0.3$, 2-tailed test).

(ii) The effect of positional asymmetry. The tests for the effect of positional asymmetry revealed (a) no significant effect of asymmetry on performance for transformation Id ($p > 0.1$, 2-tailed test) and (b) a highly significant effect of asymmetry on performance for transformation

Table 3.1. Coefficients used in the tests for the effects of separation and asymmetry in the analysis of Experiments 3.4 and 7.1.

Position combination	aa	ab	ac	ad	ae	bb	bc	bd	cc
Separation	0	1	2	3	4	0	1	2	0
test (i) c_i	-13	-4	5	14	23	-13	-4	5	-13
Asymmetry	4	3	2	1	0	2	1	0	0
test (ii) c_i	23	14	5	-4	-13	5	-4	-13	-13

Pi ($p < 0.001$, 2-tailed test).

These results provide strong support for the conclusions drawn from Experiment 3.1: performance for identical patterns is strongly affected by the distance between the patterns and is not affected by the symmetry of the positions of the patterns with respect to the point of fixation; performance for pairs of patterns related by point-inversion is strongly dependent on the symmetry of the pattern positions, and is not affected by the distance between the patterns.

The significance of these results will be discussed in the next chapter.

4. SCHEMES FOR THE INTERNAL REPRESENTATION AND THE PROCESSES WHICH CAN ACT ON IT: EXPLANATORY POWER AND PREDICTIONS

In the experiments reported in the previous Chapter, subjects made 'same-different' judgements about pairs of patterns which were related by certain transformations. The patterns were presented in spatial arrangements which varied in the symmetry of the pattern positions with respect to the point of fixation, and in the distance between the two patterns. Both symmetry and separation were shown to influence 'same' detection performance, as follows.

Identical patterns. 'Same' detection performance is markedly reduced by increasing separation of the patterns, whereas positional symmetry has no effect.

Reflected or point-inverted patterns. 'Same' detection performance is reduced by increasing asymmetry of the positions of the patterns, whereas separation has no effect (at least when the positions of the patterns are symmetric).

90° rotated patterns. Performance depends on the mean eccentricity of the patterns, that is, the mean distance of the pattern positions from the point of fixation.

There follows a review of the two types of schemes for the internal representation (IR) and the processes which act on it. The schemes were introduced in Chapter 1. It will be suggested that neither type of scheme can explain the results of Chapter 3, and, to support this suggestion, it will be necessary to make some assumptions about the relationship between the processing of the IR and the detection performance that one would expect to find experimentally. Only the most natural and parsimonious assumptions will be made; a model will be rejected if it does not *naturally* explain the results.

4.1 Transformation schemes

These were described in Section 1.2.1. Transformation schemes are characterized by three general assumptions:

(a) the IR is 'pointillistic' or pictorial in nature;

(b) the internal processes which can operate on the IR are certain families of compensatory transformations;

(c) the 'sameness' of two stimuli is detected if one of these families of transformations can bring the IR of one of the stimuli into coincidence with the IR of the other.

In order to make predictions about detection performance, a further assumption must be made concerning the 'cost', in lost detection performance, of objective transformations of the stimulus. There are three reasonable assumptions which could be made about this. First, one could reasonably assume that the operation of compensating for an objective transformation is a noisy operation, and that the total effect of noise should increase with increasing size of objective transformation for which the system has to compensate. 'Size' is intended to mean a natural measure of the scale of the transformation, so that the size of a translation is measured by the shortest distance between its end points, and a rotation through 5° is *not* regarded as a rotation through 365°. The noisier the operation, the more likely it is to fail, so one would predict a decrease in detection performance with increasing size of objective transformation. Second, one could assume instead that there is a time limit on the operation of the scheme, after which the operations must fail. If the time taken to perform the compensatory transformations increases with the size of the objective transformation, one would again predict a decrease in detection performance with increasing size of objective transformation. Finally, if one assumes that all the internal transformations which are available can be successfully completed noiselessly and with no time limit (or even that all transformations take the same time) one predicts *no* effect of the size of the objective transformation.

There are no natural assumptions, however, which would lead one to predict that detection performance should *increase* with the size of objective transformation: it is not reasonable to suppose that 'same-detection' is facilitated by having to do more to the IR. Therefore it will be assumed that the cost, in lost detection performance, of objective transformation is either zero or increases with size of transformation.

How well are these predictions fulfilled by the results described at the start of this chapter? For identical patterns the observed decrease in detection performance with distance fits quite well with a transformation scheme with assumed positive costs of size of transformation, where here the transformation is translation in the plane; for reflected or point-inverted patterns the observed detection performance does not fit with such a scheme, for the following reasons. Under a transformation scheme, one would suppose that objective transformations such as reflection or point inversion would be compensated for by an appropriate family of rotations in 3- or 2- dimensions, respectively. Given that the detection performance for identical patterns showed increasing costs with increasing translation, one would expect detection performance for reflected or point inverted patterns to fall off with the separation of the patterns in the plane, since translations would have to be combined with the rotations to bring the IRs into coincidence. Also, the 'sameness' of 180° planar rotated (point inverted) patterns should under no conditions be easier to detect than that of 90° rotated patterns, given the assumption of non-negative cost of objective transformation. Since neither of these latter predictions is fulfilled by the data described in Chapter 3, transformation schemes clearly do not explain the results.

4.2 Structural schemes

These were described in Section 1.2.2. Structural schemes are characterized by two general assumptions:

(a) the internal representation specifies certain local features in the stimulus and the spatial relations obtaining between these features;

(b) the 'sameness' of two stimuli is evaluated by the extent to which their structural descriptions concur.

It has also been suggested (Foster & Mason, 1979) that an inversion operation, in which all relations are relabelled with or reinterpreted as their opposites, can be applied to the IR, thus explaining the high observed detection performance for point-inverted patterns.

One of the advantages cited in favour of structural schemes is that the IR can be invariant to stimulus position, by virtue of its relational structure. So one would expect that there should be no effect of pattern separation on the detection of 'sameness' of pairs of patterns which are identical, or which are related by reflection or point inversion. In other words there are no natural reasons to expect the observed effects of positional symmetry or separation on the basis of position independent structural schemes.

If one allows the structural IR to contain position information (most schemes make no commitment as to whether or not the IR contains such information), one can make assumptions which are very similar to those made in the preceding section, predicting the effects of separation on detection performance for identical patterns, but failing to predict the separation-independent performance for reflected or point-inverted patterns.

Hence structural schemes also fail to explain the results of Chapter 3.

4.3 Position information in the internal representation

Since neither structural nor transformation schemes can naturally predict the results of Chapter 3, an alternative scheme which is consistent with these results will now be developed. It will first be necessary to consider what the results can tell us about the way in which position information is expressed in the IR, since it follows from the arguments in the preceding sections that the IR, whatever its form, must contain some position information. An indication of the form of this information can be seen in the 'same' detection performance for reflected or point-inverted patterns. Although detection performance for these patterns does not depend on the separation of the patterns, it does depend on the symmetry of their positions with respect to the point of fixation. Therefore, if the IR does contain position information, it seems reasonable to hypothesise that the position of the stimulus is expressed with respect to the point of fixation. This would explain the importance of symmetry about this point.

In the scheme set out below, it will be assumed that the IR contains position information which is defined relative to the point of fixation, although it could be suggested that position is defined in the IR relative to the centre of the display, and the fact that the fixation point was always positioned there led to the artifactual importance of the fixation point. The logic of the arguments which follow is not affected if it is the case that the position of the stimulus is expressed with respect to some perceived centre of the world other than the fixation point; the important element of the argument is that position must be expressed with respect to *some* point, be it the point of fixation, the centre of the display, or any other reference point.

4.4 A new scheme for the internal representation and its processes

A new scheme will now be proposed. The scheme makes a compromise between transformation and structural schemes, and can explain the results of Chapter 3 and some of the observations which were described in Section 1.3.1. The scheme also makes some predictions which are tested in later experiments.

The scheme specifies the nature of the IR and the processes which can act on the IR, as follows.

Internal representation. Patterns are assumed to give rise to IRs consisting of collections of elements specifying (a) local pattern features, (b) the horizontal and vertical spatial relations obtaining between these features, and (c) the position of the pattern with respect to the point of fixation. Attneave (1968) has suggested a similar scheme in which the stimulus is represented with respect to separate local and global axes. This IR is an extention of the usual structural IR to include special position information, with the restriction of permitted spatial relations to those specifying horizontal or vertical relationships. This restriction allows the operation of reflection to be performed in a single step, as will be described below. The restriction also means that the scheme gives rise to certain counter-intuitive predictions, and is therefore vulnerable to disproof (see Section 1.1 and below).

Operations on the IR. It is assumed that there are two types of operation which can be performed on the IR.

(i) any individual element of the IR can be modified, but only in a progressive, continuous fashion.

(ii) All the elements of a given kind in the IR can be relabelled in a single step. The renaming (or reinterpreting) nature of this operation means that it can only be applied to the representation as a whole, not to single elements of the IR.

The 'sameness' of two stimuli is evaluated by the extent to which their IRs concur, after combinations of operations (i) and (ii) have been applied. The likelihood of a successful match is assumed to depend on the extent of modification required and on the number of different operations needed to bring the two IRs into coincidence. This last assumption will be discussed below.

The type of operation defined in (i) is intended to characterize the continuous property of the observed data. For example, continuous modification of the position component in the IR is equivalent to a compensatory translation; as suggested in Section 4.1, continuous transformation is compatible with the observed smooth decrease in detection performance with increasing separation of identical stimuli.

The type of operation defined in (ii) is intended to use the discrete nature of the IR to perform discrete transformations of the whole IR. For example, relabelling all horizontal relations by their opposites is equivalent to a compensatory reflection about a vertical axis (and also, in some cases, a translation as described below). This global relabelling of elements is intended as a reinterpretation of the IR; certain elements in the IR acquire new meanings because the rules for the interpretation of the elements are changed.

4.5 Explanatory power of the proposed scheme

The scheme set out above concerns the IR and the processes which can be performed on it. The scheme contains no explicit commitment about how these entities are used to make decisions concerning the 'sameness' of objects. The strategy adopted above of making the simplest and most natural assumptions to relate schemes to detection performance will be applied to the present scheme. The assumptions will be:

(i) the cost of continuous modification is proportional to the amount of modification required;

(ii) the cost of a global relabelling (or reinterpretation) of sets of elements in the IR is a constant, independent of the number of elements to be relabelled;

(iii) the order in which the operations are applied to the IR is random, but operations closest to the identity are performed first.

Some motivation for these assumptions will be given in Section 4.6.2.

Within the new scheme, the results reported in Chapter 3 may be interpreted as follows.

Identical patterns. Pairs of identical patterns, differing only in position, are detected as 'same' by continuous modification of the position component in the IR of one of the patterns, until the IRs of the two coincide. Increased pattern separation requires more modification before the match can be achieved, and so reduces the 'same' detectability of the patterns.

Point inverted patterns. Pairs of patterns which are positioned symmetrically with respect to the point of fixation, and which are related by point inversion, are detected as 'same' by relabelling or reinterpreting all those elements that specify spatial direction or sense in the IR of one of the patterns. Each of these elements is assigned the opposite significance so that, for example, the feature-relation 'above' becomes 'below', and the component '1° to the left of the fixation point' becomes '1° to the right the fixation point'. By virtue of the original symmetrical positioning of the patterns, this brings the two IRs into coincidence (since the relabelled position component of one is equivalent to the original position component of the other). If the two patterns are not symmetrically positioned with respect to the point of fixation, this operation does not bring the two IRs into coincidence, because their position components are still different. In this case the 'sameness' of the stimuli is less detectable because further modification of the position component in one of the IRs must follow to bring the two IRs into coincidence.

Reflected patterns. Pairs of patterns related by reflection in a vertical line are detected as 'same' in a similar way. Elements specifying horizontal direction or sense in the IR of one of the patterns are assigned the opposite significance, so that the feature-relation 'left of' becomes 'right of' and the component '0.5° to the right of the fixation point' becomes '0.5° to the left of the fixation point'. Feature relations such as 'above' and components such as '1° below the fixation point' are unaffected. As in the case of point-inverted patterns, if the positions of the patterns are horizontally symmetric with respect to the point of fixation, this operation brings the two IRs into coincidence. If the positions are not symmetric, further modification of the position component must follow before a match can be achieved, making the detection of the 'sameness' of these patterns more difficult. A similar operation could be used for detecting the 'sameness' of patterns related by reflection in a horizontal line.

90° rotated patterns. The scheme implies no specific ability to detect the 'sameness' of pairs of patterns related by rotations through angles not close to 0° or 180°. The way in which these patterns might be processed will be discussed in Section 4.8.

Similarity of mirror images. The scheme can explain the results of some of the studies discussed in Chapter 1. In that chapter it was concluded that the two patterns in a mirror image pair are in some sense most similar when:

(i) one pattern is taken into the other by a reflection, that is, no translation is involved;

(ii) the point of fixation lies on the mirror axis.

When both (i) and (ii) are true, the IRs of the two patterns in a mirror image pair can be brought to coincidence by a single global relabelling, as for reflected patterns above. When either (i) or (ii) is not true, further modification of the position component in the IR of one of the patterns must follow, which reduces the similarity of the two patterns.

4.6 Same-different processing

There are two questions which might be raised about the above scheme. First, why should the visual system have a special ability to report the 'sameness' of reflected or point-inverted patterns? Second, why, if these abilities are available, should they fail to report 'sameness' on some occasions? Some speculations concerning these questions follow.

4.6.1 Why reflections and inversions?

There are reasons why symmetry might be important to the visual system; half the information in a symmetric pattern is redundant, and could therefore be dispensed with, if special coding procedures for symmetric patterns were introduced (Barlow and Reeves, 1979). On the other hand, mirror reversals do not occur very frequently in nature, nor do inversions. The most likely answer to the first question is that the ability to report the 'sameness' of reflected or inverted patterns is not *required*. Rather, given the proposed nature of the IR, these abilities exist as a result of the economy of representation rather than because of a special requirement for reflection or inversion invariance.

4.6.2 Why do the operations fail?

There could be many reasons why these operations sometimes fail.

To make it plausible that they should sometimes fail, some 'same-different' processing models will now be considered.

In simple 'same-different' reaction time tasks, it takes considerably longer for subjects to report that two stimuli are 'different' than to report that they are 'same' (Nickerson, 1965, 1967). It has been suggested that the reason for this is that there are two separate mechanisms involved; a fast, parallel-processing 'same' reporter and a slow, serial-processing 'different' reporter (Bamber, 1969). It has been suggested further that the reason the 'same' operation is faster is that noise is likely to cause spurious 'different' responses to 'same' stimuli but is unlikely to cause spurious 'same' responses to 'different' stimuli; the 'different' reporter therefore has to double check the stimulus before reporting (Krueger, 1978). The stimuli for these experiments are usually a pair of upright letters, and so the 'same' requirements are somewhat simpler than those in the present study.

Suppose that similar logic were to apply to the present scheme. Assume that a 'same' reporter searches for a way in which the two stimuli are 'same', while a 'different' reporter searches for differentiating features. The answer 'same' or 'different' is determined by whichever reporter gives a definite answer first. Because the 'same' reporter has to search through a number of operations, it can sometimes take longer than the 'different' reporter to give an answer, and because the 'different' reporter searches only for differentiating features, it can sometimes report 'different' for stimuli which are in fact 'same'. If the 'same' reporter goes through its operations in a random order, but performing first those nearest to the identity, we have a scheme which sometimes fails, and fails most often on those stimuli which require the most operations to bring their IRs into coincidence.

It is not suggested that this is the precise way in which the operations are carried out in the visual system; many similar arguments could be put forward. It *is* suggested, however, that it is quite reasonable to propose operators which can fail on some occasions.

4.7 Predictions of the proposed scheme

Given the proposed scheme and the restrictions placed on the IR and processes, it is possible to make some testable predictions about potential experiments.

4.7.1 Oblique and other effects in the 'same' detection of reflected patterns

In the proposed scheme, the local relations between the features are restricted to those specifying horizontal or vertical relationships in order to enable the exclusive relabelling of the horizontal relations, or the exclusive relabelling of the vertical relations, to achieve a reflection operator. It follows that although it is simple to detect the 'sameness' of (i) pairs of patterns related by reflection in a vertical line which are symmetrically positioned to either side of the point of fixation and (ii) pairs of patterns related by reflection in a horizontal line which are symmetrically positioned above and below the point of fixation, it should be more difficult to detect the 'sameness' of pairs related by reflection in an oblique line, whatever the positional symmetry of the arrangement. This is because the IR contains no oblique relations, which means that an operation equivalent to compensatory reflection in an oblique lines is not simple.

Since the proposed reflection operator acts only on one set of relations (either horizontal or vertical), it follows that the ability to detect the 'sameness' of pairs of patterns related by reflection in a vertical line should not depend on their vertical positions, as long as these are the same for both patterns. A similar argument applies for pairs related by reflection in a horizontal line and their horizontal positions.

Three predictions may be made about the ability to detect 'sameness' in pairs of patterns related by reflections:

(i) if the patterns are positioned symmetrically (horizontally or vertically) about the fixation point, the highest 'same'-detection performance will occur when the reflection axis is perpendicular to an imaginary line joining the centres of the patterns;

(ii) if the patterns are positioned symmetrically about the fixation point and the line joining the centres of the patterns is oblique, the 'same' detection performance for a reflection axis perpendicular to the line will be poorer than if the line were horizontal or vertical;

(iii) the ability to detect the 'sameness' of pairs of patterns related by reflection in a vertical line will be independent of their vertical positions, if these are the same for both patterns.

Effect (i) is well known; reports concerning effect (ii) differ, but agree that performance for oblique axes is poorer than that for a vertical axis (Corballis & Beale, 1976; Corballis & Roldan, 1975; Mach, 1885; Rock & Leaman, 1965).

Neither of the effects (ii) or (iii) are predicted for point-inverted patterns. Effect (ii), an oblique effect, is not predicted since the relabelling operation equivalent to point inversion can be successfully applied to the IR independent of the orientation of the line joining the patterns, as long as the pattern arrangement is point symmetric; effect (iii), an insensitivity to vertical position, is not predicted since the relabelling operation equivalent to point inversion requires *point* symmetry to bring the IRs of the patterns into coincidence (the operation equivalent to reflection, as is implicit in the above prediction, requires only *line* symmetry). For pairs of patterns related by point inversion, then, an oblique effect is not predicted, whereas a sensitivity to vertical position *is* predicted.

For identical patterns, no oblique effect and no sensitivity to vertical displacement are predicted, since 'same' detection performance for these patterns is hypothesised to depend only on the separation of the pattern positions.

Experiments testing these predictions are reported in Chapter 5.

4.7.2 Non-invertability of local pattern features

The predictions of the previous section follow from the restrictions that the proposed scheme places on the types of *relation* that can exist in the IR. Further predictions can be made as a result of the restrictions placed on the types of *operation* that can be performed on the IR. In Section 4.4, two types of operation were proposed. These were (i) global reinterpretation of element types and (ii) local continuous modification of individual elements. These operations, it is suggested, fit in with the results reported in Chapter 3, and observations by other workers. There are, however, some alternative explanations which could be offered. For example, one could suggest that the ability to detect the 'sameness' of reflected or point-inverted patterns is mediated by a non-structural IR in which the relations between the features are not encoded; such IRs might be compared on the basis of stimulus features alone. If the system were equipped with inversion and reflection operators which act only on these features, then one could explain the ability of the system to detect the 'sameness' of these patterns. It is therefore important in the verification of the proposed scheme to demonstrate that the operations (i) and (ii) defined above are the only operations that can be performed on the IR.

One way of demonstrating this is to devise a stimulus transformation which would not affect the ability to detect 'sameness' if non-structural matching as described above were possible, but which would greatly reduce this ability if exclusively the operations in (i) and (ii) were available. Two examples of such transformations will be described in Chapter 8. In the first transformation, the positions of the features in the stimulus are modified, but the features themselves are unchanged. In the second transformation the features in the stimulus are inverted, but the positions of the features within the stimulus are unchanged. Such transformations of the stimulus can be performed only if the features and the positions of the features are known; in this example the stimulus is generated using a selection of 'synthetic' features positioned at random within a stimulus pattern.

If 'same' detection were achieved by non-structural feature-matching, both these transformations would yield stimuli which would appear very similar to the original. In contrast, if 'same' detection were achieved using only the operations in (i) and (ii) above, many relations would have to be modified by the operation in (ii) to compensate for either of these objective transformations, and it would be difficult to detect the 'sameness' of the original and transformed stimuli.

An experiment which tests the effects of the above types of complex transformation is reported in Chapter 8.

4.8 'Same' detection for rotated patterns

The proposed scheme does not explain the fact that although detection performance for pairs of patterns related by 90° rotation is on average lower than that for pairs related by the other transformations, this performance is nevertheless higher than chance level (see Section 1.3.4). Some explanation must be given for the ability of subjects to recognize these patterns on at least some occasions. Sutherland (1973) has indicated that recognition of 90° rotated stimuli does not occur unless it is facilitated by special features in the stimulus. Special stimulus features may have occurred in the experiments reported in Chapter 3. Subjects reported that it was easy to detect 'sameness' in certain pairs of patterns which were elongated or which had a distinctive feature such as a spur or a cluster. These patterns were easily detected as 'same' in any position combination or after any of the pattern transformations. These reports suggest that detection of certain stimuli might be achieved by a more direct non-structural feature matching process which depends on the existence of 'strong' features in the stimulus for its operation. If this were the case, direct matching might be most important for the detection of 'sameness' of pairs of patterns related by 90° rotation, since there is in theory no other efficient way in which the system can detect the 'sameness' of these patterns. One might therefore expect performance for these patterns to be strongly dependent on the particular pattern used since the random nature of the patterns implies a variability in the number of 'strong' features in the patterns. One might expect less pattern dependence in the detection of 'sameness' of patterns related by the other transformations, since these patterns do not in theory require 'strong' patterns features as a *prerequisite* for 'same' detection.

An experiment concerning pattern specific effects is reported in Chapter 6.

5. EFFECTS OF DISPLAY ORIENTATION, POSITION OFFSET, AND REFLECTION AXIS ORIENTATION IN THE DETECTION OF 'SAMENESS' OF REFLECTED PATTERN PAIRS

In the previous Chapter a scheme was put forward to explain the effects of positional symmetry and separation on the visual comparison of patterns. The scheme gave rise to three predictions about the visual comparison of a pair of patterns of which one is a reflected version of the other:

(i) if the patterns are positioned symmetrically (horizontally or vertically) about the fixation point, the highest 'same' detection performance will occur when the reflection axis is perpendicular to an imaginary line joining the centres of the patterns;

(ii) if the line joining the centres of the patterns is oblique, the 'same' detection performance for a reflection axis perpendicular to the line will be poorer than if the line were vertical or horizontal;

(iii) for pairs of patterns positioned to either side of the fixation point, the ability to detect the 'sameness' of patterns related by reflection in a vertical line will be independent of their vertical positions, if the vertical positions are the same for both patterns. Together with these predictions, it was suggested that performance for identical patterns should show no oblique effect and no dependence on vertical position, and that performance for point inverted patterns should depend on vertical position and show no oblique effect.

To test these predictions, three experiments were performed. In Experiment 5.1, subjects performed 'same-different' comparisons of pairs of patterns presented simultaneously, one pattern to the left and one to the right of the point of fixation. The patterns could be related by one of six transformations: identity, reflection in one of four different orientations of reflection axis, or point inversion. In Experiment 5.2, the patterns were presented simultaneously on opposite sides of the point of fixation, and the line joining the centres of the patterns could take one of four orientations. The transformations relating the patterns were identity, point inversion, or reflection in an axis perpendicular to the line joining the centres of the patterns. In Experiment 5.3, pairs of patterns were presented simultaneously, one pattern to each side of the fixation point, and the vertical distance of the patterns from the fixation point was either zero, or equal to the horizontal distance. The transformations relating the patterns were identity, point inversion or reflection in a vertical line.

In all the experiments the subject's task was to decide if the two patterns in a pair were 'same', taking into account possible rotations or reflections, or 'different'.

5.1 Experiment 5.1

5.1.1 Methods

Subjects. The subjects were two male students in the Department of Communication and Neuroscience, and two female visitors to the Department. All were aged between 23 and 26 years, had normal or corrected-to-normal vision, and all, except the author, were unaware of the purpose of the experiment.

The display. The display was as described in Section 2.1. Fixation was aided by a large cross consisting of two computer generated white lines, 3.125° long; a fixation spot was superimposed on the centre of the cross. The spot was displayed throughout each presentation; the cross was extinguished at the start of each trial.

Stimuli. The stimuli were random dot patterns as described in Section 2.3.

Pattern positions. In each trial two patterns appeared simultaneously, one centred 0.5° to the left of the fixation point, the other 0.5° to the right.

Pattern transformations. There were six possible transformations relating the patterns in each 'same' pair. These were:

Id : the two patterns were identical;

Mi_{θ}: one pattern was obtained from the other by reflection in a line oriented at an angle θ clockwise from the vertical, θ taking the values 0°, 45°, 90° or 135°;

Pi : one pattern was obtained from the other by point inversion, that is, planar rotation through 180°.

For 'different' pairs, two independent patterns were generated.

A fresh pattern or pair of patterns was generated for every trial.

Instructions and presentation sequence. The instructions to the subject and the sequence of events in a presentation were as described in Sections 2.4 and 2.5.

Experimental design. In each run, each of the six 'same' pattern transformations occurred four times, and there were twenty four 'differents' in a run. Each subject performed twelve runs in one session. For the purpose of balancing the order of the pattern transformations, each run was split into four sections of twelve trials. Within each section, the order of the pattern transformations was chosen randomly but balanced for order and carry-over effects over runs (see Section 2.6). For each subject, four random sequences were generated at the start of the experiment and were permuted each run to implement the balanced design.

5.1.2 Results

Fig. 5.1 shows 'same-different' pattern discrimination performance in Experiment 5.1. The discrimination index d' (see Section 2.7) is plotted against pattern transformation.

The d' data are pooled over subjects. A Chi-squared test on individual data (see Section 2.7) revealed no significant differences between subject's absolute performance levels ($p > 0.2$). Contrast tests on the average data and also on individual data yielded the following results.

(i) *Best axis of reflection.* 'Same' detection performance for patterns related by reflection in a vertical line (Mi_0) is higher than that for patterns related by the other reflections (Mi_{45}, Mi_{90}, and Mi_{135}) as predicted ($p < 0.001$ for average data and for all individual data, 2-tailed tests).

(ii) *No significant difference between Id and Mi.* There is no significant difference between performance levels for identical patterns (Id) and for those related by reflection in a vertical axis (Mi_0) ($p > 0.7$ for average data, $p > 0.3$, $p > 0.4$, $p > 0.1$, $p > 0.9$ for individual data, 2-tailed tests.)

(iii) *Significant difference between Id and Pi.* Performance for identical patterns (Id) is significantly higher than that for patterns related by point inversion (Pi), for average data only ($p < 0.01$, average data, $p < 0.05$, $p > 0.4$, $p > 0.2$, $p > 0.05$ for individual data, 2-tailed tests).

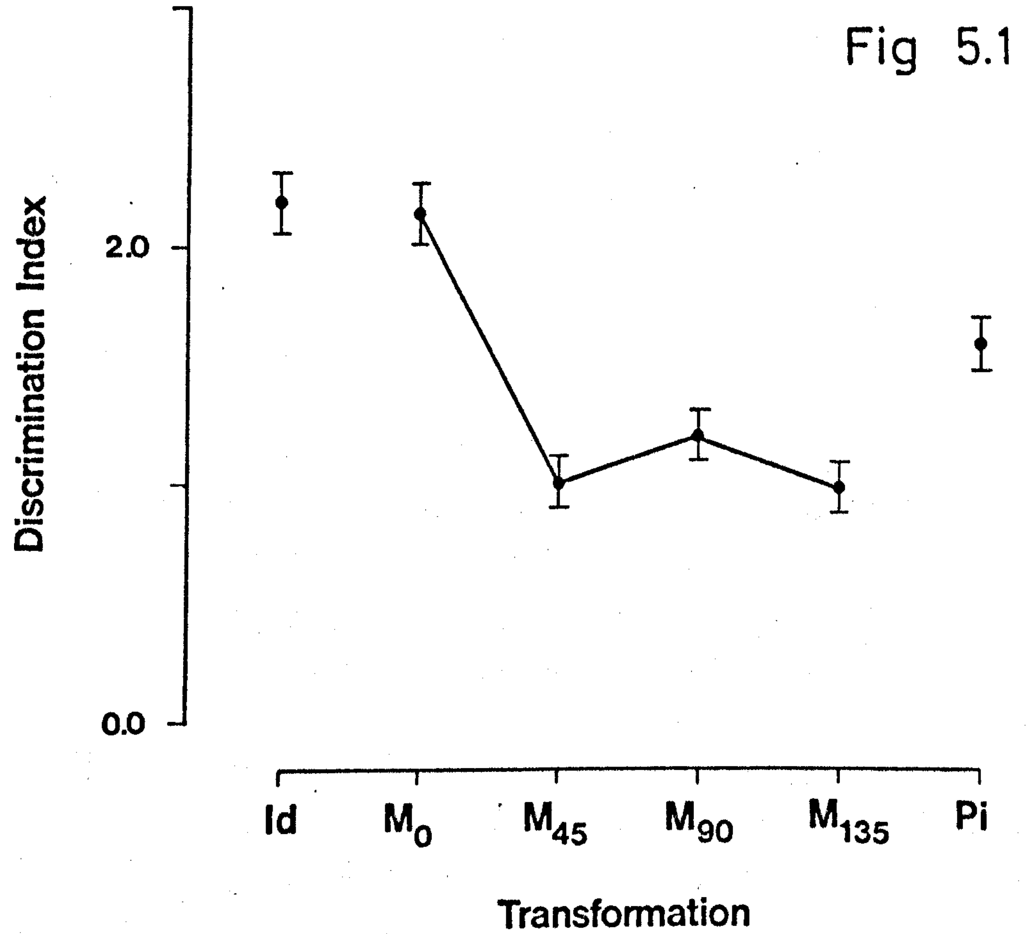


Figure 5.1. 'Same' detection performance in Experiment 5.1. The pooled discrimination index d' is plotted against pattern transformation. The pattern transformations are as follows. Id: the patterns are identical; MI_θ: the patterns are related by reflection in a line oriented at θ° clockwise from the vertical; PI: the patterns are related by point inversion.

(iv) *Significant difference between Pi and non-vertical mirror axes.* Performance for patterns related by point inversion (Pi) is significantly higher than that for patterns related by reflection in a non-vertical line (Mi_{45}, Mi_{90}, Mi_{135}) ($p < 0.001$, average data; $p < 0.5$, all subjects, 2-tailed tests).

(v) *No significant difference between Mi_{90} and Mi_{45}, Mi_{135}.* There is no significant difference between performance for patterns related by reflection in a horizontal line (Mi_{90}) and performance for those related by reflection in an oblique line (Mi_{45}, Mi_{135}) ($p > 0.1$, average data, $p < 0.01$, $p > 0.2$, $p > 0.9$, $p > 0.6$, individual subjects, 2-tailed tests).

The result that the highest performance for reflected patterns occurs when the mirror axis is vertical bears out prediction (i) of Section 4.7.1 and is consistent with the results of Foster and Mason (1979). The prediction is not counterintuitive, so the result is not surprising. It was necessary, however, to establish that this is the 'best' axis before continuing with Experiment 5.2.

5.2 Experiment 5.2

5.2.1 Methods

Subjects. The subjects were five male students in the Department of Communication and Neuroscience, and one female visitor to the Department. All were aged between 23 and 27 years, had normal or corrected-to-normal vision, and all except the author were unaware of the purpose of the experiment.

The display. The display and aids to fixation used in this experiment were the same as those used in Experiment 5.1.

Stimuli. The stimuli used in this experiment were 'normalized' random dot patterns. These were standard dot patterns, which were generated using the method described in Section 2.3, and which were subsequently normalized by scaling their horizontal and vertical extents so that the horizontal separation of the extreme pair of dots was 0.5°, and the vertical separation of the extreme pair of dots was 0.5°. This had the effect of making the patterns appear more uniform in shape, thus preventing the use of certain inappropriate decision strategies. In pilot experiments with non-normalized patterns, the task was so easy that all scores for the Id and Mi stimuli (which will be described below) were very close to 100% and were therefore statistically unreliable (see Section 5.3.2 for a discussion of the importance of normalization).

Pattern positions. In each trial, two patterns appeared simultaneously, one positioned 0.5° from the fixation spot in one of four directions, the other positioned 0.5° from the fixation spot in the opposite direction. The four directions, 0°, 45°, 90°, and 135°, measured clockwise from the horizontal, defined four 'display orientations', so that in 'display orientation' 0° the patterns were side by side, and in 'display orientation' 90° one pattern was above the other.

Pattern transformations. There were three possible transformations which could relate the two patterns in each 'same' pair:

Id: the two patterns were identical;

Mi: one pattern was obtained from the other by reflection in the perpendicular bisector of the imaginary line joining the pattern positions;

Pi: one pattern was obtained from the other by point inversion, that is, planar rotation through 180°.

For 'different' pairs, two independent patterns were generated.

A fresh pattern or pair of patterns was generated for every trial.

Instructions and presentation sequence. The instructions to the subject and the sequence of events in a presentation were as described in Sections 2.4 and 2.5.

Experimental design. In each run, every display orientation (0°, 45°, 90°, 135°) occurred twice with each of the three 'same' pattern transformations (Id, Mi, Pi), and six times with differents, so that a run consisted of 24 'sames' and 24 'differents'. Each subject performed 24 runs over a period of several days. Because of the number of treatments used in this experiment, the design is complex. For the purpose of balancing the order of the display orientations each run was split into 12 sections of 4 trials each. These represented the four runs of three 4 x 4 'mini-experiments' in the order row 1 of experiment 1, row 1 of experiment 2, row 1 of experiment 3, row 2 of experiment 1, row 2 of experiment 2, etc. For each 'mini-experiment' the order of the display orientations was chosen randomly but balanced for order and carry-over effects over 'mini-runs'. For each subject, three new random sequences were generated for every run, and permuted *within* the run.

The sequence of the pattern transformations occurring with a given display orientation was random. For this, four new sequences were generated every run.

5.2.2 Results

Fig. 5.2 shows 'same-different' pattern discrimination performance in Experiment 5.2. In each graph the discrimination index d' (see

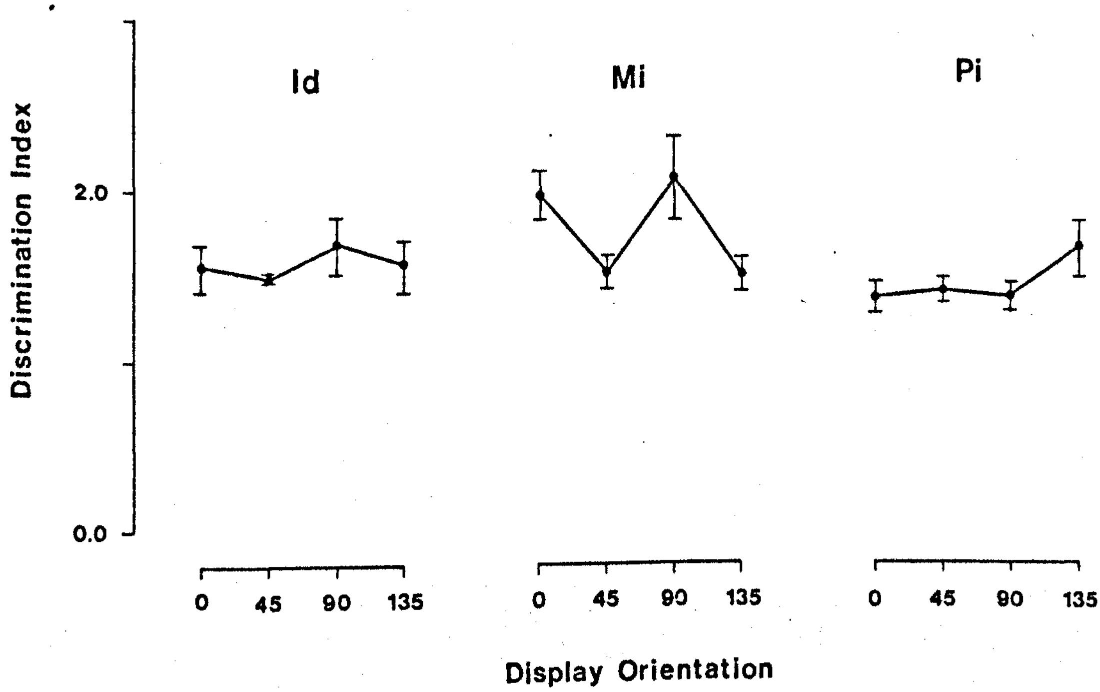


Fig 5.2

Figure 5.2. 'Same' detection performance in Experiment 5.2. Each graph shows the pooled discrimination index d' plotted against display orientation for one of the pattern transformations Id, Mi and Pi. The display orientations and pattern transformations are described in the text.

Section 2.7) is plotted against 'display orientation' (0°, 45°, 90° 135°) for one of the pattern transformations (Id, Mi, Pi).

The d' data are pooled over subjects. The error bars shown on the graphs are the standard errors in the means of the d' levels, calculated after subtraction of each subject's overall performance level from his d' scores. Standard errors are used since a Chi-squared test on the individual data yielded significant differences between subjects after allowing for each subject's overall performance level ($p < 0.01$); the method of estimating variances described in Section 2.7 would lead to an underestimate of the variance.

Contrast tests on the data (see Section 2.7) yielded the following results.

(i) *Oblique effects.* There is no significant oblique effect for transformations Id or Pi ($p > 0.2$, $p > 0.9$, respectively, 1-tailed tests); there is a highly significant oblique effect for transformation Mi ($p < 0.001$, 1-tailed test). Oblique effects are tested by contrasting performance on display orientations 0° and 90° with performance on orientations 45° and 135°. The tests are 1-tailed since an 'oblique effect' requires that performance on the latter is poorer than performance on the former.

(ii) *Relative levels.* Performance for transformation Mi in display orientations 0° and 90° is significantly higher than overall performance for transformation Id ($p < 0.001$, 2-tailed test). There is no significant difference between performance for transformation Mi in display orientations 45° and 135° and overall performance for transformation Pi ($p > 0.4$, 2-tailed test).

These results confirm the prediction that the ability to detect the 'sameness' of patterns related by reflections should show an oblique effect, and that such an effect should not occur for pattern pairs which are identical or which are related by point inversion.

5.3 Experiment 5.3.

5.3.1 Methods

Subjects. The subjects were 9 male students in the Department of Communication and Neuroscience. All were aged between 21 and 27 years, had normal or corrected-to-normal vision, and all, except the author, were unaware of the purpose of the experiment.

The display. The fixation aids and display were the same as those used in Experiment 5.1.

Stimuli. The stimuli were normalized random dot patterns as described in Section 5.2.1.

Pattern positions. In each trial two patterns appeared simultaneously; the distance of each of the patterns from the point of fixation was always 1.0°. There were two combinations of positions used in this experiment:

C: (centred) the pattern were positioned on a horizontal line through the fixation point, one to the left, the other to the right;

O: (offset) the positions of the patterns were offset. In this case the imaginary line joining the fixation point to the pattern position was 45° from the horizontal; the patterns were either both above or both below the level of the fixation point, one to the left and one to the right.

Instructions and presentation sequence. These were as described in Sections 2.4 and 2.5.

Experimental design. In each run, both of the position combinations occurred 3 times with each 'same' pattern transformation and 9 times with 'differents' so that a run consisted of 18 'sames' and 18 'differents'. Each subject performed 12 runs in one session. In the offset position combination, 'O', the patterns were either both above or both below the fixation point. In every two runs the 'O' combination occurred six times with each pattern transformation, three times as 'above' and three times as 'below'. The order of the pattern transformations and position combinations was chosen randomly but balanced for order and carry-over effects over runs (see Section 2.6). For each subject, a random sequence was generated at the start of the experiment and was permuted for each run to implement the balanced design.

5.3.2 Results

Fig. 5.3 shows 'same-different' pattern discrimination performance in Experiment 5.3. The discrimination index d' (see Section 2.7) is shown for both position combinations and for each of the pattern transformations.

The d' data are pooled over subjects. Chi-squared tests on individual data (see Section 2.7) revealed (i) significant differences between subjects' absolute performance levels ($p < 0.01$) and (ii) no significant differences between subjects after allowing for each subject's overall performance level ($p > 0.9$).

Contrast tests (see Section 2.7) revealed that the offset in position combination O (compared with combination C) cause a small, non-significant increase in performance for both transformations Id and Mi ($p > 0.5$, $p > 0.1$, respectively, 2-tailed tests) and a

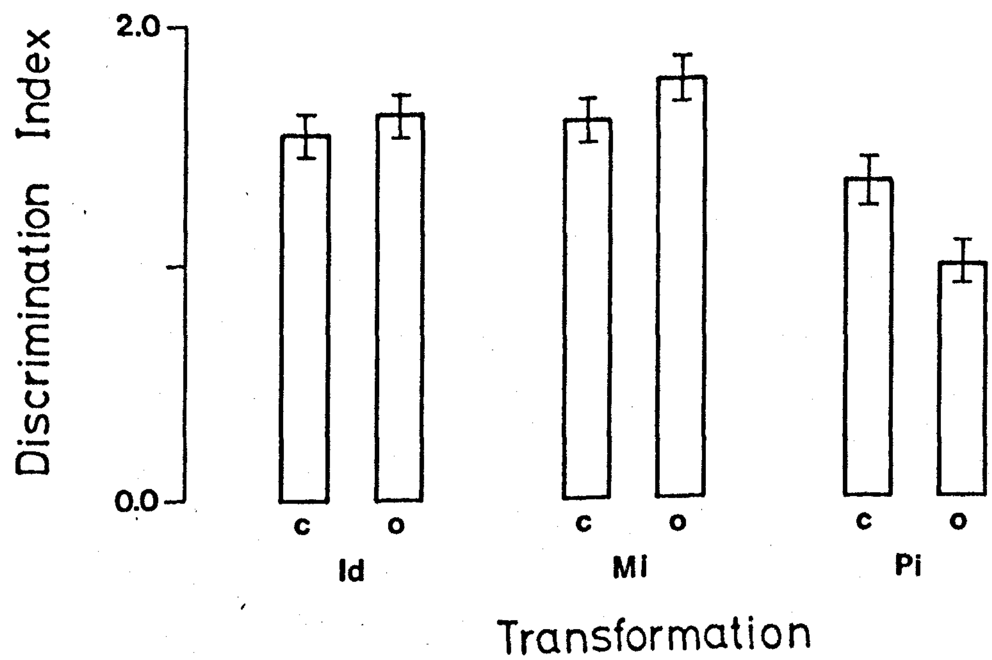


Fig 5.3

Figure 5.3. 'Same' detection performance in Experiment 5.3. The pooled discrimination index d' is shown for the centred (C) and offset (O) position combinations for each of the pattern transformations Id, PI, and MI.

significant *decrease* in performance for transformation Pi ($p < 0.01$, 2-tailed test). Also, the change in performance between combinations C and O for transformation Pi is significantly different from that for transformation Id and Mi ($p < 0.01$, 2-tailed test).

These results fulfill the predictions of the scheme proposed in Chapter 4; the ability to detect the 'sameness' of pairs of patterns related by reflection in a vertical line is not affected by a vertical displacement of the patterns; the ability to detect the 'sameness' of pairs of patterns related by point-inversion is significantly reduced by vertical displacement.

5.4 Summary and discussion of Experiments 5.1, 5.2 and 5.3.

We can conclude from these experiments that:

(i) for two patterns positioned symmetrically on each side of the point of fixation, the highest 'same' detection performance for reflected pairs occurs when the axis of reflection is perpendicular to the imaginary line joining the pattern positions;

(ii) when the axis of reflection *is* perpendicular to this imaginary line, 'same' detection performance is higher when the line is vertical or horizontal than when the line has an oblique orientation;

(iii) no such orientation effects are found to exist for 'same' detection performance on pairs of patterns which are identical or which are related by point inversion;

(iv) 'same' detection peformance for pairs of patterns related by reflection in a vertical line is independent of the vertical displacement of the pairs from the point of fixation, whereas performance for pairs related by point inversion is reduced by vertical displacement.

In the scheme proposed in Chapter 4, pairs of stimuli related by reflection are detected as 'same' by reversal of all horizontal or vertical relations in the internal representation of one of the stimuli. The oblique effect for reflected pairs is consistent with the notion that there are no oblique relations in the IR, which means that the detection of the 'sameness' of oblique pairs is not a simple operation. The fact that *no* oblique effect occurs for pairs of patterns related by point inversion is consistent with the notion that these stimuli are detected as 'same' by inversion of *all* the relations in the internal representation of one of the stimuli. This operation is simple providing that the positions of the patterns are symmetrical with respect to the point of fixation, independent of the orientation of the imaginary line joining the pattern positions.

Further, the model suggests that reversal of horizontal or vertical relations also requires the reversal of the position information in the internal representation. The fact that the 'best' reflection axis in (i) above is vertical is consistent with this idea; when the patterns are positioned side-by-side, symmetrically about the point of fixation, reversal of the horizontal position information brings the position represented in the two IRs into coincidence; reversal of the vertical position information in the IR does not affect the *positions* represented in the IRs which are *different*, so that reflection in a non-vertical line is not a simple operation; the reversal of relations must be followed by continuous modification of the position information in the IR to achieve a match.

The scheme suggests that the operator used to detect the 'sameness' of patterns related by reflection in a vertical line should have no effect on the vertical position information in the IR, whereas the operator used for point inverted patterns reverses both horizontal and vertical position information. Thus when the positions of the two patterns are offset as in display orientation O of Experiment 5.3, the relabelling operation equivalent to reflection in a vertical line will bring the positions represented in the IRs of the two patterns into coincidence, whereas the relabelling operation equivalent to point inversion does not bring the positions represented in the IRs into coincidence; further modification of the position component in one of the IRs must follow to achieve a match.

There are two problems concerning Experiment 5.2 which must be considered before this model can be fully accepted:

(i) why is performance for oblique reflections not considerably lower than that for point inversion?

(ii) why is stimulus normalization necessary?

These problems are discussed in the next section.

5.4.1 High performance on obliques

If the system is not specifically equipped to respond to pairs of patterns related by oblique reflections, why is 'same' detection performance for such stimuli as high as that for pairs related by point inversion, to which the system *is* specifically equipped to respond? One might speculate that this arises for a number of reasons.

First, it is possible that there is more than one way in which mirror related pairs can be detected as 'same'; one might speculate that apart from the inversion operator described above, mirror pairs can be detected as 'same' by a direct comparison of the stimulus features nearest the axis of reflection Julesz, 1971; Bruce & Morgan, 1975. Such a matching process would not work for pairs of identical patterns or pairs of patterns related by point inversion since no 'same' features are directly opposite each other in the former case, and in the latter case only those near the centre-centre line are 'same'. Such a process would increase, by a fixed increment, performance on pairs related by reflection, independent of orientation.

A second more plausible hypothesis is that the system uses display orientation cues to realign the internal coordinate system and to subsequently re-encode the stimuli with exclusively oblique relations, so that the relabelling equivalent to an oblique reflection becomes a simple operation on the modified IR (such reorientations of the internal frame of reference have been previously suggested by Rock and Leaman, 1963; Attneave and Olson, 1967; Attneave, 1968; Rock, 1973). This reorientation and re-encoding operation would have costs in terms of discrimination performance, so that the 'sameness' of pairs of patterns related by oblique reflections would be more difficult to detect than that of pairs related by vertical or horizontal reflections. The very high detection performance for patterns related by horizontal or vertical reflections might then be explained by the fact that only one set of relations (horizontal or vertical) in the IR has to be reversed, compared with the two sets which have to be inverted for pairs of patterns related by point inversions.

Independent of these considerations, the critical result is that the mirror pairs exhibit the oblique effect predicted by the proposed scheme; this is an effect that is not predicted by transformation or structural schemes.

5.4.2 Normalization of the stimuli

The stimuli used in Experiments 5.2 and 5.3 were normalized random dot patterns (see Section 5.2.1). These stimuli were selected because, in pilot studies using standard dot patterns, performance for all the display orientations was high; this was as a result, it will be suggested, of certain inappropriate decision strategies. The introduction of normalization caused a decrease in performance for patterns related by reflection in the oblique axes and a small decrease for identical patterns; there was no change in performance for patterns related by point inversion or by horizontal or vertical reflection. The observed oblique effect cannot be an artefact of the normalization or of the axes of the normalization, since no oblique effect was found for identical or point-inverted patterns.

It is suggested that the normalization of the stimuli prevents the subject from making judgements based on the rule that "equality of width of the patterns measured perpendicularly to the line joining the pattern positions means that the patterns are (probably) 'same' ", because normalization makes the widths of 'different' patterns the same. Thus the removal of a cue to 'sameness' impedes 'same' detection for patterns related by reflections in an oblique axis, and has little or no effect for the other transformations. One reason why this cue should be particularly important in these experiments is that no 'same' patterns whose perpendicular widths could differ were presented, so that the subject could employ this strategy with no risk of missing 'same' pattern pairs. It should be noted that the strategy of reporting 'different' if the perpendicular widths of the patterns are different would affect only the false alarm rate for a given display orientation, thus affecting the measured performance for all the transformations in that orientation. In order to have a differential effect on performance for transformation Pi versus transformation Mi, say, normalization must inhibit the strategy of reporting 'same' if the perpendicular widths are the same, and that strategy must be of use for detecting only those patterns which are related by oblique reflections.

One might conclude that the ability to detect 'sameness' of patterns related by point inversion is a result of width matching also; this might explain the drop of performance with rotation angle up to 90° and its subsequent rise around 180° (see Sections 1.2.2 and 1.3.4). This argument can be rejected on the following grounds:

(i) detection performance for pairs of patterns related by point inversion is not affected by the introduction of normalization;

(ii) in Experiment 5.1, detection performance for pairs of patterns related by vertical reflection or by point inversion is higher than that for patterns related by horizontal reflection, even though the perpendicular widths of the patterns in these three cases are the same;

(iii) width matching is not capable of explaining the results of Experiment 3.1.

5.4.3 Conclusions

The results of Experiments 5.1, 5.2 and 5.3 give strong support to the assertions that the relations in the IR specify only horizontal and vertical relationships, that the IR contains position information which is defined with respect to the point of fixation, and that there are two distinct types of operation which can be performed on this IR. This conclusion is not weakened by the considerations explored in Sections 5.4.1 and 5.4.2; these are minor problems of interpretation.

6. PATTERN SPECIFIC EFFECTS

In all the experiments reported so far, the approach has been to average responses over the largest possible sample from the universe of random dot patterns, with the underlying assumption that it is possible to average out any effects that are related to specific patterns. However, in Section 4.8 it was suggested that there may be some effects of 'strong' features in particular patterns, especially when the distribution of the positions of the dots is non-uniform. Suitable 'strong' features might be, for example, a spur or a cluster in the pattern, or elongation of the pattern. The experiment reported in this Chapter is designed to test whether performance is dependent on the individual patterns, and, if so, to discover how the ability to detect 'sameness' in a *given* pair of patterns depends on the various transformations which can relate the patterns.

In this experiment, instead of a fresh pattern or pair of patterns being generated for every trial, a fixed set of twenty-one patterns was generated, one set for each subject. Each pattern was presented ten times in each of three 'same' arrangements (Id, Ro and Pi), and each of the 210 possible 'different' pairs of patterns was presented three times. As usual, subjects were required to decide if the two patterns were 'same', taking into account possible rotations, or 'different'. From this experiment, one can decide, for a given transformation (Id, Ro, or Pi), whether the probability of a successful detection of 'sameness' is a function of the transformation alone, or whether it also depends on the individual pattern. A problem in the design of this experiment is that in the design described above, pattern effects are confounded with subject effects. That is, if there turn out to be differences between the results of two experiments on different subjects, these may have arisen either because the subjects are different, or because of the different selection of patterns. The alternative is to choose one fixed set of patterns for use with all the subjects. If this second design were to be used, the experimenter would have no way of testing whether, by chance, the selection of twenty-one patterns is unrepresentative of the population of patterns. This was the overriding consideration in the choice of the first design.

In Section 4.8, it was suggested that 'strong' features can enable the use of a special form of non-structural feature matching. If this were true, one might expect that the number of correct reports of 'sameness' for a given transformation would depend both on the pattern and on the transformation applied to it, as follows.

(i) It is hypothesised that the ability to detect the 'sameness' of pairs of patterns related by rotation through 90° (transformation Ro) depends exclusively on the existence of 'strong' features in the stimulus. Therefore one might predict that performance (that is, the number of correct 'same' responses) for these pattern pairs should show a large amount of variability between patterns, corresponding to the variability in number of 'strong' features in the patterns.

(ii) It is hypothesised that the ability to detect the 'sameness' of patterns related by point inversion, that is, planar rotation through 180° (transformation Pi), does not necessarily *require* 'strong' features in the pattern, although these might sometimes be effective. Most patterns of this type will be detected as 'same' by the inversion operation described in Sections 4.4 and 4.5. Therefore one might predict that if there is any variability between patterns in performance for transformation Pi, performance for a given pattern will be correlated with that for transformation Ro. That is, the same pattern effects are expected for both Pi and Ro, but the effects will be less for Pi than for Ro.

(iii) Similar arguments apply for identical pattern pairs (transformation Id), although performance for these patterns has been so high in previous experiments that it seems unlikely that 'strong' features will make any significant contribution to performance.

6.1 Experiment 6

6.1.1 Methods

Subjects. The subjects were five male students and one female student in the Department. All were aged between 21 and 26, had normal or corrected-to-normal vision, and all, except the author, were unaware of the purpose of the experiment.

The display. The display was as described in Section 2.2. Fixation was aided by a circle of dots with a fixation dot in the centre. The fixation dot was displayed throughout each trial; the circle was extinguished at the start of each trial.

Stimuli. The stimuli were random dot patterns as described in Section 2.3. For each subject a set of 21 such patterns was generated at the start of the experiment.

Pattern positions. In each trial two patterns appeared simultaneously, one centred 0.5° to the left of the fixation point, the other centred 0.5° to the right.

Pattern transformations. For each 'same' pair, one pattern was selected from the set of 21 patterns, and one of three transformations was applied to it to define the second pattern. The transformations were:

Id: the second pattern was identical to the first;

Ro: the second pattern was obtained from the first by planar rotation through 90°;

Pi: the second pattern was obtained from the first by point inversion, that is, planar rotation through 180°.

For each 'different' pair, two patterns were selected from the set of 21 patterns, without replacement.

Experimental design. In each run, each 'same' transformation (Id, Ro, Pi) occurred 7 times, and there were 21 'different' pairs. The order of the transformations Id, Ro, Pi and 'Diff' was chosen randomly but balanced for order and carry over effects over runs (see Section 2.6). Ro occurred equally often as a clockwise or anticlockwise rotation.

The occurrence of patterns was as follows. In three runs every pattern appeared three times in a 'same' pair, related once by each of the transformations Id, Ro and Pi. The order of combinations of transformation and pattern was random. In ten runs each pattern appeared once in a 'different' pair with each other pattern, in random order. There were 30 runs in each experiment, so each pattern appeared a total of 90 times; 10 times in a 'same' pair related by each of Id, Ro and Pi; and 60 times in 'different' pairs.

To ensure that there were no systematic effects arising from the orientations of the patterns, each pattern was presented in each of 90 orientations spaced in 4° angles occurring in random order, so that no pattern occurred in precisely the same orientation more than four times, and in most cases, only once. This happens four times if by chance four orientations of a given pattern, spaced in 90° intervals, are

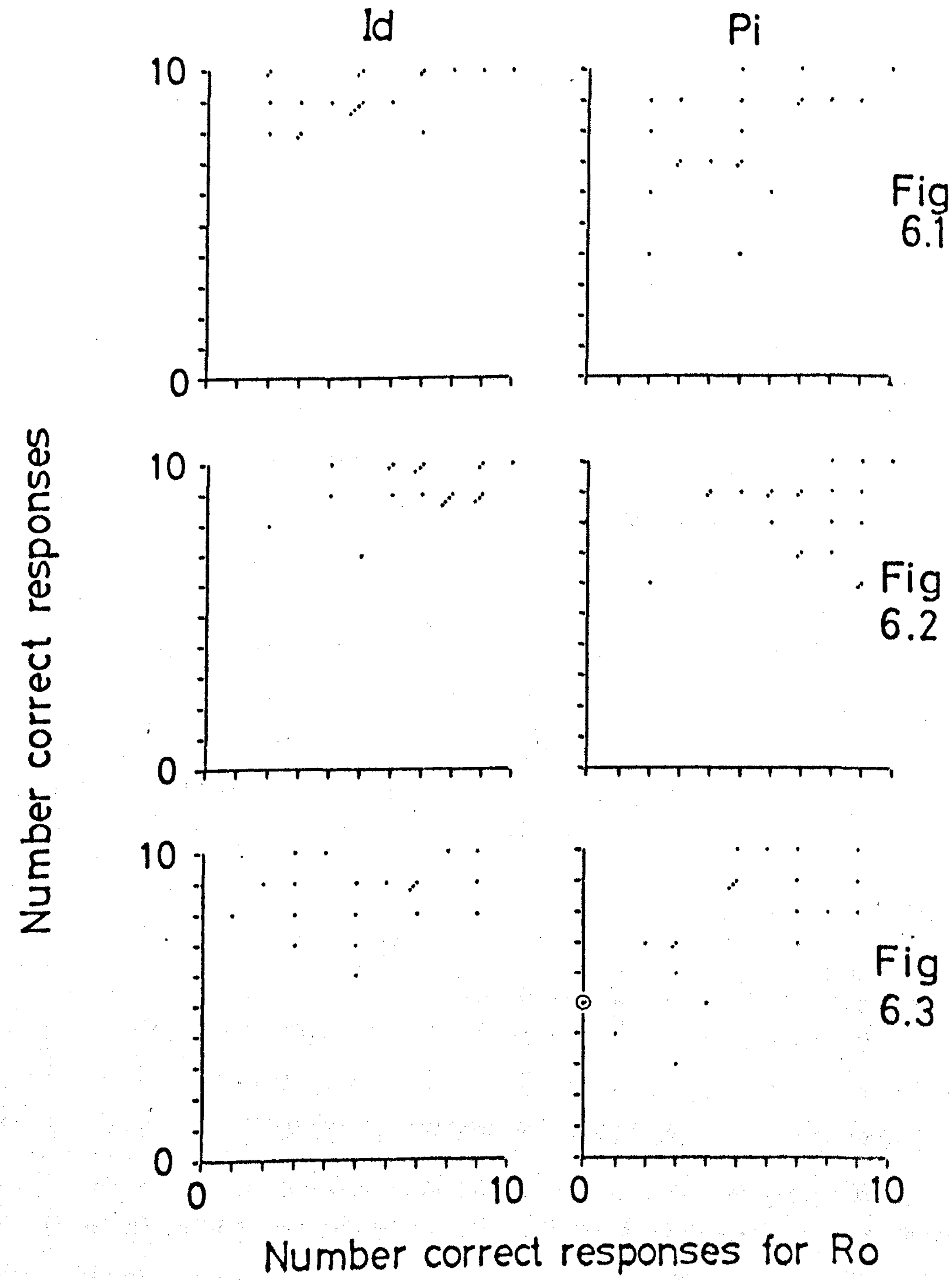
are transformed respectively by Id, Ro (+90°), Pi, and Ro (-90°). This will not, of course, happen often.

The design, although complicated, is the simplest which fulfills the following requirements. To ensure that subjects cannot learn the form of patterns and subsequently use their ability to *recognize* just one of the patterns in a pair to make the judgement 'same' or 'different' (rather than judge by comparison between the two patterns), it is necessary that the same set of patterns be used for 'sames' and for 'differents'. For similar reasons it is also required that each pattern must appear equally often with each of the 'same' transformations (Id, Ro and Pi); that each pattern must appear equally often in 'different' pairs; and that each possible combination of different patterns appears equally often (there are 210 combinations here).

Instructions and presentation sequence. The instructions to the subject and the sequence of events in a presentation were as described in Sections 2.4 and 2.5.

6.1.2 Results

The results of Experiment 6 are displayed in Figs. 6.1 - 6.6. The two graphs in each figure are scatter plots. For each stimulus pattern, a point is plotted on each of the graphs. The y-coordinate of the point represents the total number of correct 'same' responses to the pattern when presented in a pair related by the given transformation (Id on the left hand graph, Pi on the right hand graph). The x-coordinate in both graphs represents the number of correct 'same' responses when the pattern is presented in a pair related by transformation Ro. Since each pattern was presented ten times under



Figures 6.1 to 6.6. Results of Experiment 6. Each figure is a scatter plot of the number of correct 'same' responses made by a given subject; for each point in the left-hand plot the y-coordinate represents the number of 'same' responses to a given pattern under transformation Id, and the x-coordinate represents the number for that pattern under transformation Ro; in the right-hand plot the y-coordinate represents the number of correct 'same' responses under transformation Pi, and the x-coordinate again represents that for transformation Ro.

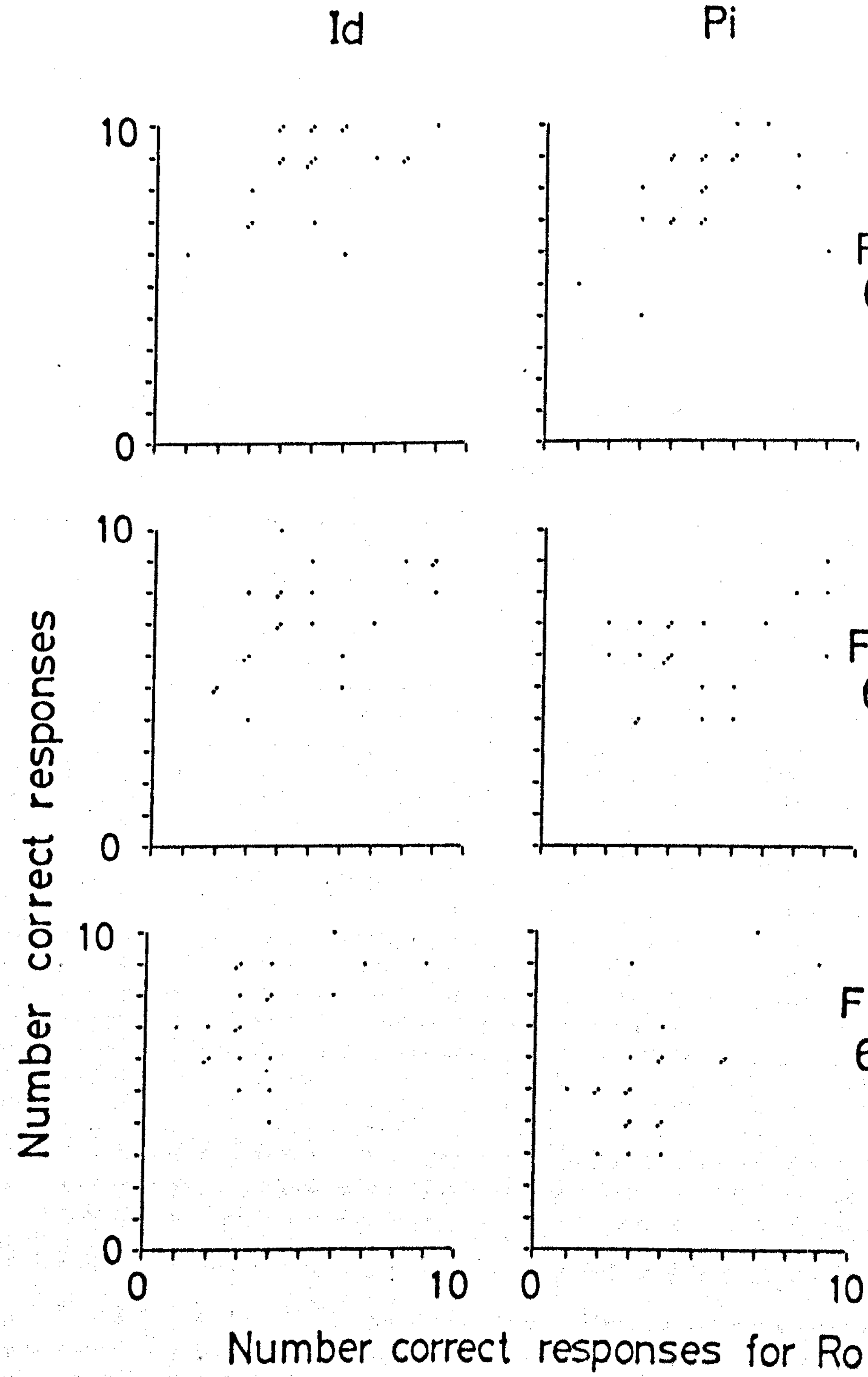


Fig 6.4

Fig 6.5

Fig 6.6

each 'same' transformation, the range of the number of correct 'same' responses is zero to ten. Figs. 6.1 to 6.6 show the results for subjects JK, RS, MM, RW, PF, and SG respectively.

6.1.3 Statistical analysis of the results of Experiment 6

A number of statistical tests were used in the analysis of the results of this experiment; these are described in Section 6.3. The tests yielded the following results. (For all the significance levels reported below, the results are given in the subject order JK, RS, MM, RW, PF, SG; this is the order that the results appear in Figs. 6.1 - 6.6.)

(i) *Performance levels averaged over all patterns.* Performance (that is, the number of correct 'same' responses) for transformation Id is significantly higher than that for transformation Pi for five out of six subjects ($p < 0.01$, $p < 0.05$, $p < 0.05$, $p < 0.05$, $p > 0.05$, $p < 0.001$, 2-tailed tests), and performance for transformation Pi is significantly higher than that for transformation Ro for all subjects ($p < 0.001$, $p < 0.05$, $p < 0.001$, $p < 0.001$, $p < 0.05$, $p < 0.01$, 2-tailed tests).

(ii) *Homogeneity of probability of correct response between patterns.*

(a) Performance for transformation Id is not significantly dependent on pattern ($p > 0.7$, $p > 0.7$, $p > 0.5$, $p > 0.05$, $p > 0.2$, $p > 0.2$, 1-tailed test; an overall test combining the tests on individual subjects also revealed no significant pattern dependence for transformation Id, $p > 0.2$, 1-tailed test).

(b) For two subjects performance for transformation Pi is significantly dependent on pattern ($p < 0.05$, $p > 0.2$, $p < 0.01$, $p > 0.05$, $p > 0.7$, $p > 0.1$, 1-tailed tests; an overall test combining the tests on individual subjects revealed a significant pattern dependence for transformation Pi, $p < 0.01$, 1-tailed test).

(c) For four subjects performance for transformation Ro is significantly dependent on pattern ($p < 0.01$, $p < 0.05$, $p < 0.01$, $p > 0.2$, $p < 0.05$, $p > 0.2$, 1-tailed tests; an overall test combining the tests on individual subjects revealed a highly significant pattern dependence for transformation Ro, $p < 0.001$, 1-tailed test).

(d) For all subjects, performance summed over all transformations is significantly dependent on pattern ($p < 0.01$, for all subjects, 1-tailed tests).

(iii) *Correlation between performance for various transformations.*

(a) For four out of six subjects, performance for transformation Id on a given pattern is not significantly correlated with that for transformation Ro ($p > 0.1$, $p > 0.1$, $p > 0.1$, $p > 0.05$, $p < 0.01$, $p < 0.05$, 1-tailed tests; these significance levels correspond to correlation coefficients of 0.29, 0.28, 0.20, 0.37, 0.50, 0.39 respectively).

(b) For three out of six subjects performance for transformation Pi on a given pattern is significantly correlated with that for transformation Ro ($p < 0.5$, $p > 0.3$, $p < 0.001$, $p < 0.05$, $p > 0.1$, $p > 0.05$, 1-tailed tests; these significance levels correspond to correlation coefficients of 0.49, 0.09, 0.70, 0.47, 0.29, 0.36 respectively).

(c) For all subjects the slope of the regression line of performance for transformation Id on that for transformation Ro is significantly different from unity ($p < 0.01$, $p < 0.01$, $p < 0.01$,

$p < 0.5$, $p < 0.01$, $p < 0.05$, 2-tailed tests; these significance levels correspond to slopes of 0.29, 0.19, 0.10, 0.50, 0.40, 0.50 respectively); for five out of six subjects, the slope of the equivalent regression of Pi on Ro is significantly different from unity ($p < 0.05$, $p < 0.01$, $p < 0.05$, $p < 0.05$, $p < 0.01$, $p > 0.2$, 2-tailed tests; these significance levels correspond to slopes of 0.50, 0.21, 0.56, 0.41, 0.28, 0.71, respectively).

(iv) *Learning or 'unlearning' effects.* No subjects displayed any significant learning or 'unlearning' effects, where 'unlearning' is defined as a decreasing probability of correct 'same' response to a given pattern with successive exposures of that pattern ($p > 0.7$, $p > 0.9$, $p > 0.4$, $p > 0.9$, $p > 0.3$, $p > 0.4$, 1-tailed tests).

6.2 *Summary and discussion of results of Experiment 6.*

The results of Experiment 6 can be summarized as follows:

(i) relative performance levels for the three transformations are as expected from previous observations (see Section 1.3.4 and Chapter 3);

(ii) the ability to report 'sameness' of pairs of patterns related by transformations is not solely a function of the transformation; there is also a significant effect of pattern. This effect is most noticeable for transformation Ro; pattern dependence is less marked for transformation Pi and is small or non-existent for transformation Id. Pattern effects are similar for transformations Pi and Ro in the sense that a pattern that gives rise to relatively high performance (number of correct 'same' responses) for transformation Ro will give rise to slightly increased performance for transformation Pi.

The predictions listed in the introduction to this Chapter are borne out by these results. For all subjects there are effects which are pattern specific, and for most subjects these pattern specific effects are most marked for transformation Ro. The fact that, with some patterns, performance for transformation Ro was very low is consistent with the assertion that there is no mechanism based on structural feature matching for comparing these patterns, and that a lack of 'strong' features caused the failure of 'same' detection. We are left with the question of what are the 'strong' features which can facilitate the detection of 'sameness'. For each subject, the three patterns which gave rise to the lowest performance in Experiment 6 are reproduced in Fig. 6.7, and the three which gave the highest performance are reproduced in Fig. 6.8. Although with hindsight one could suggest that the patterns in Fig. 6.8 display more non-uniformities than those in Fig. 6.7, such judgements are at best subjective. An attempt was made to characterize the 'featurefulness' of the patterns used in the experiment by means of various techniques, such as metric structure measures (Moore, Seidl and Parker, 1975) and other specially developed measures based on the distributions of chord lengths and angles between chords in a pattern (a chord is a line joining two dots in a pattern). Unfortunately, none of these techniques yielded objective pattern measures which were significantly correlated with experimentally measured performance on the patterns. One ad hoc pattern measure did, however, appear to bear some relation to the experimentally measured performance. The measure was the kurtosis of the distribution of chord lengths in the pattern (the kurtosis is defined by $Ku = \mu_4/\sigma^4 - 3$ where μ_4 is the fourth moment of the distribution about the mean and σ is the standard deviation; kurtosis is a measure of the

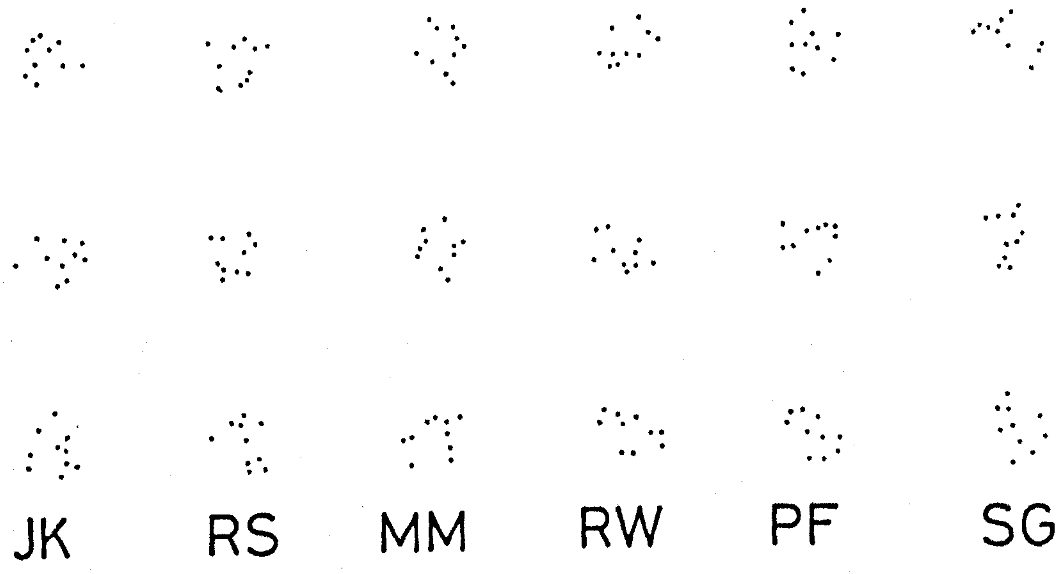


Fig 6.7

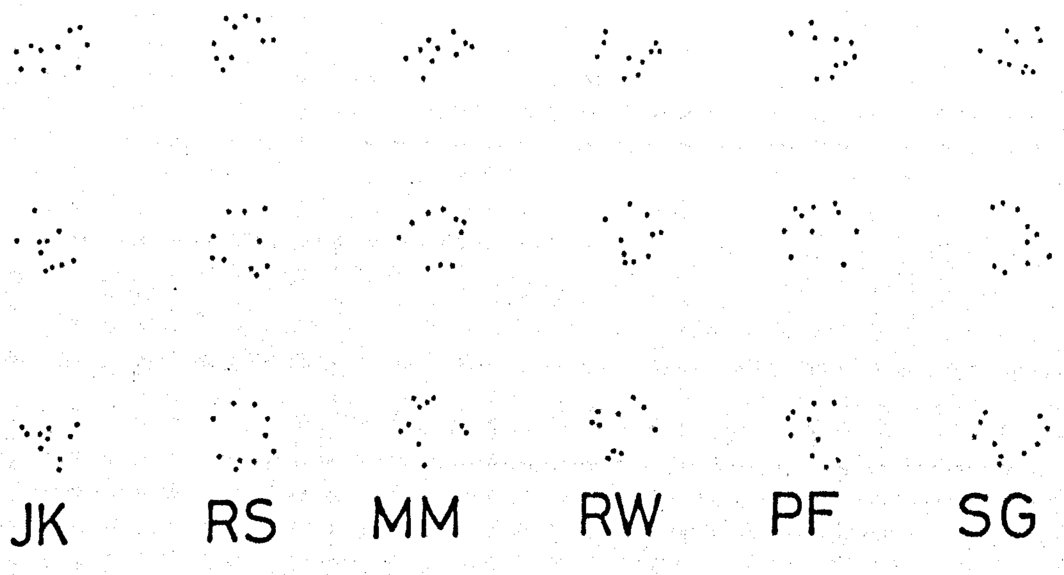


Fig 6.8

Figures 6.7 and 6.8. The three dot patterns which gave rise to the lowest number of correct 'same' responses in Experiment 6 are shown in Fig. 6.7, and those which gave rise to the highest number are shown in Fig. 6.8.

peakiness of the distribution). Although the kurtosis was not significantly correlated with performance, the difference between the average kurtosis on the top ten patterns and that on the bottom ten patterns, where top and bottom are defined by performance under transformation Ro, taken over all subjects, was significantly different from zero (Chi-squared test, $p < 0.05$). That is, patterns with a less peaky distribution of chord length are on average more easy to report as 'same' under transformation Ro than those with a more peaky distribution. This result is not easy to interpret in terms of the structure of patterns, and is reported here only to indicate that it is possible to find some association between objective pattern measures and experimentally measured performance. It was concluded that an analysis of the outcome of this experiment in terms of objective pattern measures will have to wait until an adequate mathematical theory of the structure of patterns has been developed. In the absence of objective structure measures one can draw only the limited conclusion that the ability to detect the 'sameness' of patterns related by rotation through 90° is dependent on some unknown pattern parameter which also, to a lesser extent, affects the ability to detect the 'sameness' of patterns related by point inversion. This conclusion is not inconsistent with the hypothesis that the ability to detect the 'sameness' of patterns related by 90° rotation is mediated by a direct non-structural feature matching process, which is, on average, less efficient than the processes outlined in Sections 4.4 and 4.5, and which works only in the presence of strong pattern features. For patterns related by the other transformations (Id and Pi), performance is not strongly dependent on pattern; pattern specific effects are of minor significance in the overall scheme.

6.3 Supplement to Chapter 6: Statistical tests used in the analysis of Experiment 6.

(i) Test for differences between performances, under a given pair of transformations, averaged over all patterns. For each subject, let the number of correct 'same' responses for pattern i under a given transformation j be r_{ij} where i = 1, 2, ..., 21 specifies the pattern and j = 1, 2 specifies the two transformations under test. Let the probability of a correct 'same' response (or success) under transformation j be p_j, the best estimate for which is $\hat{p}_j = \frac{r_{.j}}{210}$, where $r_{.j} = \sum_i r_{ij}$ (210 is the total number of trials for each 'same' transformation).

If we define $p = \dfrac{r_{.1} + r_{.2}}{420}$,

$$q = 1 - p,$$

$$\sigma_\Delta = \sqrt{\frac{pq}{5}}$$

and $$\hat{z} = \frac{\hat{p}_1 - \hat{p}_2}{\sigma_\Delta}$$

then under the null hypothesis that $p_1 = p_2$, $\hat{z}$ is distributed as a standard normal variable (see Hoel, 1971, Section 9.3).

(ii) Test for homogeneity of probability of 'success'. This test determines whether the probability of a correct 'same' response (success) under a given transformation (Id, Ro or Pi) is independent of the pattern to which the transformation is applied.

For a given subject and a given transformation, let R_j be the number of successes for the pattern specified by j = 1, 2, ..., 21.

Then the empirical logistic transform of R_j (see Cox, 1970, Section 5.2),

$$z_j = \log \frac{R_j + \frac{1}{2}}{n_j - R_j + \frac{1}{2}} ,$$

is approximately normally distributed with approximate variance

$$Vj = \frac{(n_j + 1)(n_j + 2)}{n_j(R_j + 1)(n_j - R_j + 1)} ,$$

where n_j, the number of trials on the j^{th} pattern, is 10.

$$\text{If } \bar{z} = \frac{1}{21} \sum_j z_j \quad , \quad \text{let } \rho_j = \frac{z_j - \bar{z}}{\sqrt{V_j}} .$$

Under the null hypothesis that z_j = constant, $j = 1, 2, \ldots, 21$, (equivalent to the hypothesis that p_j, the probability of success on the j^{th} pattern, is constant over j), ρ_j is a standard normal variable and $\chi^2 = \sum_j \rho_j^2$ is distributed as Chi-squared with 20 degrees of freedom.

A similar test can be performed on the total number of successes on a given pattern under all the 'same' transformations, using $n_j = 30$, and also the results can be combined over subjects by summing χ^2 and summing the degrees of freedom.

(iii) Test for correlation. This test investigates whether there is any correlation over patterns between performance on two transformations. The Spearman rank correlation coefficient (Winkler and Hays, 1975, Section 12.19) is calculated as follows. Each pattern is ranked using as a score the number of successes under one of the transformations in the comparison, and then ranked using the number of successes under the other. The correlation coefficient is then defined by

$$r_s = 1 - \frac{6\sum_j d_j^2}{n(n^2 - 1)} ,$$

where $j = 1, 2, \ldots, 21$ specifies the pattern,

d_j is the difference between the two rankings of each pattern j,

and $n = 21$ is the number of patterns.

The significance of the correlation is tested using the Fisher r to z transformation (Winkler and Hays, 1975, Section 10.3)

$$z = \tfrac{1}{2} \log \frac{1 + r_s}{1 - r_s} .$$

Under the null hypothesis that there is no correlation, z will be approximately normally distributed with standard deviation $\sigma = \frac{1}{(n - 3)^{\frac{1}{2}}}$.

To test the slope of the best fitting line to the data in the scatter plots of Figs. 6.1 - 6.6, a linear regression was carried out on the logistic transforms defined in test (ii) above. Logistic transforms were used to place more weight on the centre of the range, as the statistical reliability of points close to 0 or to 10 is very poor. If z_{ij} is the logistic transform of the number of successes on the i^{th} pattern under the j^{th} pattern transformation, the best estimate for the slope of the linear regression of performance for transformation 1 on performance for transformation 2 is

$$b = \frac{n\sum_i r_{i1}r_{i2} - \sum_i r_{i1} \sum_i r_{i2}}{n\sum_i r_{i2}^2 - (\sum_i r_{i2})^2}$$

(see Hoel, 1971, Section 7.5). The test of interest concerns whether the slope of the regression line is significantly different from unity, that is, whether *all* the pattern effect under Ro is manifest in performance under Pi. Under the null hypothesis that b = 1, the statistic

$$t = (b - 1)(s_2 \sqrt{n - 2})/s_{1.2}$$

is distributed as T with n-2 degrees of freedom. Here

$$n = 21 \quad ,$$

$$s^2_j = n \sum_i z^2_{ij} - (\sum_i z_{ij})^2 \quad ,$$

and $s_{1.2} = \sqrt{s^2_1(1 - r^2)}$ where r, the correlation coefficient of z_{i1} with z_{i2}, is defined by

$$r = \frac{n\sum_i z_{i1}z_{i2} - \sum_i z_{i1} \sum_i z_{i2}}{(n[\sum_i z^2_{i2} - (\sum_i z_{i2})^2]\ n[\sum_i z^2_{i1} - (\sum_i z^2_{i1})])^{\frac{1}{2}}}$$

The reason that a non-parametric correlation test was used in the first test and not in the second is that the former type of test involves no distributional assumptions. The latter type of test, however, is not possible without assuming bivariate normality.

(iv) Test for learning effects. This test is designed to determine whether there are any sequential effects in subjects' responses arising from the fact that patterns are presented more than once. Subjects may, for example, learn the shape of a particular pattern and subsequently always respond to this pattern with one particular response ('same' or 'different') independent of the second pattern in the presentation, despite the fact that this will lead to chance levels of correct responses. To test for this type of effect, one may look for either growth or decay with successive presentations of the probability of a 'same' response in a given combination of pattern with transformation. For a given pattern k, (k = 1, 2, ..., 21, specifying pattern) and transformation j (j = 1, 2, 3 specifying Id, Ro, Pi) let Y_i be the response to the i^{th} presentation of this combination of pattern and transformation (Y_i = 1 specifying 'same', Y_i = 0 specifying 'different', say, i = 1, 2, ..., 10). Let $t_1 = \sum_i Y_i$, the number of correct 'same' responses, and let $T = \sum_i iY_i$ (see Cox, 1970, Section 5.3). Under the null hypothesis that the probability of correct 'same' response shows no linear increase or decrease with i, T is approximately normally distributed with mean $E(T) = t_1m_1$ and variance

$$V(T) = \frac{t_1(n - t_1)m_2}{(n - 1)}$$

where $m_1 = \sum_i \frac{i}{n} = 5.5 \quad (n = 10)$

and $m_2 = \sum_i \frac{(i - m_1)^2}{n} = 8.25$

and thus $\rho_{jk} = \frac{T - E(T)}{(V(T))^{\frac{1}{2}}}$ is a standard normal variable.

Under the same null hypothesis, $\chi^2 = \sum_{jk}\rho_{jk}^2$ is distributed as Chi-squared with 63 degrees of freedom.

7. THE EFFECTS OF PATTERN ARRANGEMENT ON THE VISUAL COMPARISON OF DILATATED AND DILATATED-INVERTED PATTERNS.

A stimulus transformation which has not yet been considered in this study is dilatation (or magnification). Objects viewed from different distances present different sized retinal images, but they retain their identity and 'sameness' (see Section 1.3.3); that is, the visual system is equipped to compare patterns of different sizes. It was suggested in Section 1.3.3 that such a comparison is likely to be mediated by a scaling operation rather than by a size independent IR.

The scheme proposed in Chapter 4 does not give rise to clear predictions about the effects of dilatations since there are a number of ways in which size information could be incorporated in the IR, all of which are compatible with the scheme. If size information in the IR is encoded in a similar way to position information, however, the scheme proposed in Chapter 4 does suggest that any operations on the size information should be continuous (by analogy with the position component), so that one would expect a fall in 'same' detection performance with increasing size difference. Assuming that size information in the IR can be modified by such a continuous operation, a question of interest is how scaling affects the position component in the IR. There are two possibilities that are compatible with the scheme of Chapter 4:

(a) the global position component and all other elements of the IR are scaled by the size component, so that a change in the size component causes a change in the scale by which all the elements of the IR specifying distance are interpreted;

(b) the size component scales all those elements in the IR specifying distance, excluding the global position component.

In (a), magnification effectively takes place about the point of fixation, and in (b) magnification effectively takes place about the centre of the stimulus.

Experiment 7.1 was designed to investigate whether either of these schemes can predict the way in which pattern arrangement affects performance on pairs of patterns related by dilatations. Because the results of the experiment were unexpected, they were confirmed in Experiments 7.2 and 7.3.

7.1 Experiment 7.1

This experiment was designed to investigate the effects of positional symmetry and separation on 'same' detection performance for transformations Id and Pi, when these are combined with a dilatation. In each trial two patterns, one of which was twice the size of the other, were presented sequentially, each in one of the positions which were used in Experiment 3.4; that is, 1.0°, 0.5°, 0° to the left of fixation point and 0.5° and 1.0° to the right. All (non-equivalent) pairs of these positions were used in the experiment, except that, for the reasons outlined below, the larger pattern was always the more eccentric in a pair. (If we denote the positions by a,b,c,d,e respectively, and large/small patterns by upper case/lower case letters, the combinations used were Aa, Ab, Ac, Ad, Ae, Bb, Bc, Bd, Cc and their mirror equivalents Ee, Ed, Ec, Eb, Ea, Dd, Dc, Db, Cc respectively.) The subject's task was to decide if the two patterns in a pair were 'same', taking into account the size difference and possible rotations, or 'different'.

Scheme (a) above suggests that 'same' detection performance on patterns differing only in size should be best on pairs presented in position combination Ab, because modification by a factor 2 of the size component of the IR of, for example, the smaller pattern, will also modify the global position component into coincidence with the global position component in the IR of the larger pattern. For any other position combination, modification of the position component must follow the modification of the size component to bring the two IRs into coincidence. For dilatated point-inverted patterns, the model predicts that performance should be best for position combination Ad, since modifying the scale by a factor of 2 and inverting all relations brings the IR of the smaller pattern into coincidence with that of the larger pattern. For any other position combination further modification of the position component must follow the modification of the size component and the inversion of elements in the IR, in order to bring the two IRs into coincidence.

Since it assumes no interaction between size and position information scheme (b) suggests that 'same' detection performance in this experiment will show exactly the same qualitative properties as in Experiment 3.4: 'same' detection performance for pairs of patterns related by dilatation should depend mainly on the pattern separation, whereas 'same' detection performance for pairs of patterns related by dilatation *and* point inversion should depend mainly on the symmetry of the positions of the patterns with respect to the point of fixation.

Both schemes suggest that the highest 'same' detection performance will be found in position combinations for which the larger pattern is the more eccentric, so only these position combinations were included in the experiment.

7.1.1 Methods

The methods and subjects in this experiment were the same as for Experiment 3.4, with the following exceptions:

Stimuli. The stimuli were random dot patterns similar to those described in Section 2.3, but were generated in two sizes. The small patterns were generated within an imaginary disc of $0.167°$ radius, each dot subtended about $0.03°$ visual angle, and the minimum centre to centre separation of the dots was $0.05°$. The large patterns were generated in the same way and then magnified by a factor 2; however the dot size remained $0.03°$, as this was a property of the display that could not be controlled.

Pattern positions. The pattern positions and the position combinations were the same as those used in Experiment 3.4. Whenever the two pattern positions in a trial were different, the large pattern was presented in the more eccentric position.

Pattern transformations. There were two possible transformations relating the patterns in each 'same' pair:

Dil_{Id}: the large pattern was obtained from the small pattern by a magnification of 2;

Dil_{Pi}: the large pattern was obtained from the small pattern by a magnification of -2, that is, point inversion combined with a magnification of 2.

For 'different' pairs, two independent patterns were generated, one large and one small.

7.1.2 Results

Fig. 7.1 shows 'same-different' pattern discrimination performance

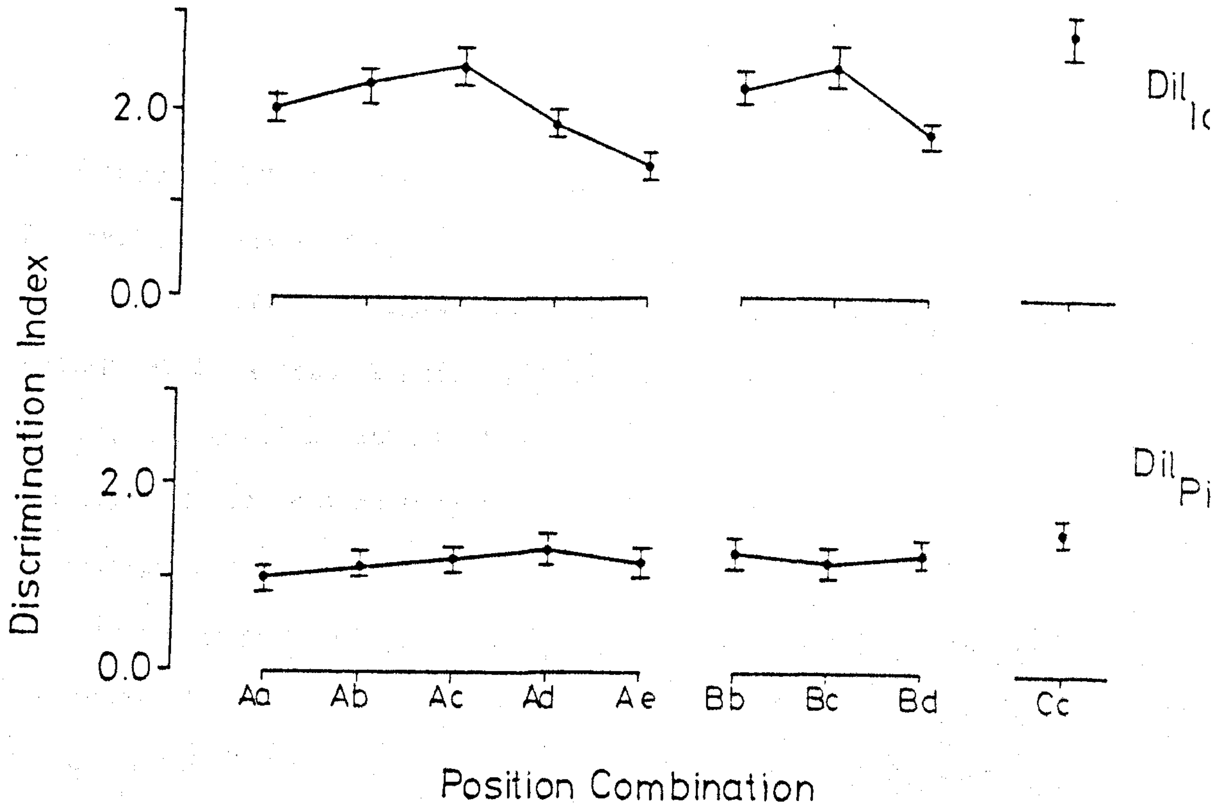


Fig 7.1

Figure 7.1. 'Same' detection performance in Experiment 7.1. Each graph shows the pooled discrimination index d' plotted against position combination.

in Experiment 7.1. In each graph the discrimination index d' (see Section 2.7) is plotted against position combination.

The upper graph shows performance for transformation Dil_{Id}, and the lower graph shows performance for transformation Dil_{Pi}.

The d' data are pooled over subjects. Chi-squared tests on individual data (see Section 2.7) revealed: (i) significant differences between subjects' overall performance levels ($p < 0.05$) and (ii) no significant differences between subjects, allowing for each subject's overall performance level ($p > 0.5$).

In the graphs in Fig. 7.1, the degree of positional asymmetry increases along positions combinations Cc, Bb, Aa, and for those points the separation of the patterns is zero. The separation of the patterns increases along position combinations Cc, Bd, Ae and for those points, the positions of the patterns are symmetric. Intermediate points are intermediate in symmetry and separation.

On the upper graph (Dil_{Id}) performance shows a large peak along points Ac, Bc, Cc, that is, when the smaller pattern is presented at the fixation point. Performance when the patterns are both presented to one side of the fixation point is considerably higher than when the patterns are presented to opposite sides. On the lower graph (Dil_{Pi}), the performance surface is nearly flat, showing only a slight increase with increasing symmetry.

The data were subjected to the tests for the effects of positional symmetry and separation described in Section 3.4.2.

(i) *The effects of separation.* The tests for the effects of separation revealed (a) a highly significant effect of separation on performance for transformation Dil_{Id} ($p < 0.001$, 2-tailed test), and

(b) no significant effect of separation on performance for transformation Dil_{Pi} ($p > 0.8$, 2-tailed test).

(ii) *The effects of positional asymmetry.* The tests for the effects of positional asymmetry revealed (a) no significant effect of positional asymmetry on performance for transformation Dil_{Id} ($p > 0.3$, 2-tailed test) and (b) a small but not significant effect on performance for transformation Dil_{Pi} ($p > 0.05$, 2-tailed test).

(iii) *The effects of the eccentricity of the small pattern.* Since there is a peak in performance on transformation Dil_{Id} at position combinations Cc, Bc, Ac, a test was performed to determine the effect of the eccentricity of the small pattern independent of the position of the large pattern. The test was derived in the same way as those described in Section 3.4.2, and the coefficients which were used are displayed in Table 7.1.

The test revealed (a) a highly significant effect of the eccentricity of the small pattern for transformation Dil_{Id} ($p < 0.001$, 2-tailed test) and (b) no significant effect for transformation Dil_{Pi} ($p > 0.1$, 2-tailed test). [Given the post hoc nature of the test, the result for Dil_{Id} should be regarded as significant, but not highly significant.]

These results do not fulfill the predictions of either of the schemes presented in Section 7.1. There is a peak in performance for transformation Dil_{Id} but this occurs in position combination Ac, not Ab, as suggested in scheme (a). The results cannot even be regarded as close: one cannot suggest that the peak is somehow shifted towards the centre, since scheme (a) also predicts that performance for position combination Ab should be far higher than that for position combination Ad, which is not found to be the case.

Table 7.1. Coefficients used in the test for the effects of the eccentricity of the position of the small pattern in the analysis of Experiment 7.1.

Position combination	Aa	Ab	Ac	Ad	Ae	Bb	Bc	Bd	Cc
Eccentricity of small pattern	2	1	0	1	2	1	0	1	0
Coefficient	10	1	-8	1	10	1	-8	1	-8

One can reject the notion that the results are an artifact of a visual acuity effect on the grounds that the minimum dot to dot separation in this experiment is identical to that used in Experiment 3.4; if the peak arose because of the inability of subjects to resolve the dots in the smaller patterns in the more peripheral positions one would expect to have found a similar result in Experiment 3.4. Another suggestion is that the *size* of the patterns is the cause of the peak, in that subjects might have difficulty in seeing peripherally presented patterns whose overall dimensions are smaller. This argument cannot be rejected on the basis of data already presented, so two further experiments were performed to control for this possibility and to investigate another experimental condition, not included in Experiment 7.1, in which a large pattern appears centrally with a small pattern presented eccentrically.

7.2 Experiments 7.2 and 7.3

Experiment 7.2 was designed to test the effects of a position combination which was not tested in Experiment 7.1, and to ensure that the results of 7.1 were robust. Four of the position combinations used in Experiment 7.1 and one further position combination were tested. In the notation used in Section 7.1, these were Aa, Ac, Ae, Cc and aC. The pattern transformations were again Dil_{Id} and Dil_{Pi}.

Experiment 7.3 was designed to test for the importance of pattern *size* when the two patterns in a pair are of the same size. In the notation of 7.1 the position combinations were AA, AE, aa and ae, and the pattern transformations were Id and Pi.

The subject's task in both these experiments was to decide if the two patterns in a pair were 'same' taking into account possible rotations (and size differences in Experiment 7.2), or 'different'.

Since these two experiments are closely linked they will be described together.

7.2.1 Methods. Experiment 7.2

The methods for Experiment 7.2 were the same as those for Experiment 7.1 with the following exceptions.

Subjects. The subjects were four male students in the Department of Communication and Neuroscience, aged between 23 and 26 years. All had normal or corrected-to-normal vision, and all, except the author, were unaware of the purpose of the experiment.

Position Combinations. The positions in which the patterns could appear in this experiment were: pattern centred 1.0° to the left of fixation point (a,A); pattern centred on the fixation point (c,C); pattern centred 1.0° to the right of the fixation point (e,E). If we use upper case letters to represent the large patterns (0.333° radius) and lower case letters to represent the small patterns (0.167° radius) the position combinations used were Aa, Ac, aC, Ae and Cc (and their mirror equivalents Ee, Ec, eC, Ea and Cc, respectively. The analysis does not distinguish between mirror equivalents).

Experimental design. In each run each position combination (Aa, Ac, aC, Ae, Cc) and its mirror equivalent occurred once with each 'same' pattern transformation (Dil_{Id} and Dil_{Pi}), and twice with 'differents', so that a run consisted of 20 'sames' and 20 'differents'. Each subject performed 20 runs over a period of several days. The order in which the patterns appeared (large or small first) was randomized. The order of the position combinations and pattern transformations was chosen randomly but balanced for order and carry-over effects (see Section 2.6). For each subject one random sequence was generated before the experiment, and permuted each run to implement the balanced design.

7.2.2 Methods. Experiment 7.3

The methods for Experiment 7.3 were the same as those used for Experiment 7.1 with the following exceptions:
Subjects. The subjects were those who performed in Experiment 7.2
Position combinations. The positions in which the patterns could occur were either 1.0° to the left of fixation point (a,A) or 1.0° to the right of the fixation point (e,E). If we use upper case letters to represent the large patterns and lower case letters to represent the small patterns, the position combinations used were AA, AE, aa and ae (and their mirror equivalents EE, AE, ee and ae respectively).
Pattern transformations. The pattern transformations used were Id and Pi as described for Experiment 3.4; in this experiment there were two sizes of pattern, but in a trial the two patterns were always the same size.
Experimental design. In each run each position combination (AA, AE, aa, ae) occurred twice with each 'same' pattern transformation (Id, Pi) and four times with 'differents', so that a run consisted of 16 'sames' and 16 'differents'. Each subject performed 16 runs in one session.

Each position combination occurred once as each of its mirror equivalent with each transformation, in every run. The order of the position combinations and pattern transformations was chosen randomly but balanced for order and carry-over effects over runs (see Section 2.6). For each subject two random sequences of 1,2,...,16 were generated before the experiment, and permuted each run to implement the balanced design.

7.2.3 Results of Experiments 7.2 and 7.3

Fig. 7.2 shows 'same-different' pattern discrimination performance in Experiments 7.2 and 7.3. In each graph the discrimination index d' (see Section 2.7) is plotted against position combination for each of the pattern transformations. The left-hand graph displays the results of Experiment 7.2, and the right-hand graph displays the results of Experiment 7.3. Performance for transformations Dil_{Id} and Id is represented by filled circles, and that for Dil_{Pi} and Pi is represented by filled squares. The dashed lines represent performance for position combination aC on the left hand graph and the dotted lines on the right-hand graph represent performance for the smaller patterns.

The d' data are pooled over subjects. Chi-squared tests on individual data (see Section 2.7) for Experiment 7.2 revealed (i) significant differences between subjects' absolute performance levels ($p < 0.05$) and (ii) no significant differences between subjects, allowing for each subject's overall performance level ($p > 0.5$). Chi-squared tests on individual data for Experiment 7.3 revealed (i) no significant differences between subjects' absolute performance levels ($p > 0.5$) and (ii) no significant differences between subjects, allowing for each subject's overall performance level ($p > 0.1$).

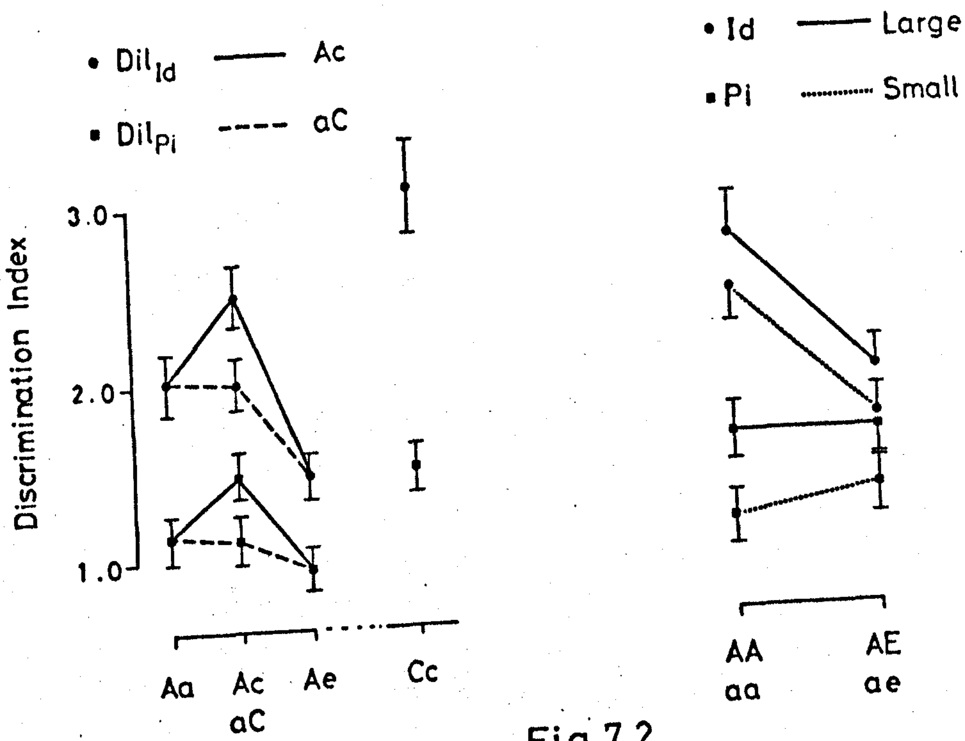


Fig 7.2

Figure 7.2. 'Same' detection performance in Experiments 7.2 and 7.3. Each graph shows the pooled discrimination index d' plotted against position combination, the left-hand graph for Experiment 7.2 and the right-hand graph for Experiment 7.3. Performance for transformations Dil_{Id} and Id is represented by filled circles and performance for transformations Dil_{Pi} and Pi is represented by filled squares. The dashed lines represent performance for position combination aC on the left-hand graph and the dotted lines on the right-hand graph represent performance for the smaller patterns. The position combinations and pattern transformations are defined in the text.

The left hand graph confirms the result that performance for position combination Ac is higher than that for Aa or Ae, but this is true of both transformation Dil_{Id} and transformation Dil_{Pi} (an effect which was not statistically significant in Experiment 7.1). Performance for transformation Dil_{Id} in position combination Aa is considerably higher than that for position combination Ae.

The right hand graph shows that the results of Experiment 7.1 cannot be explained by suggesting that the size of the patterns causes poor performance; performance for transformation Id is only slightly (and not significantly) affected by the size of the patterns.

Contrast tests on the pooled data (see Section 2.7) revealed the following:

(i) the effects of the eccentricity of the small pattern with the large pattern eccentric (position combination Ac compared with Aa and Ae): there is a highly significant effect for transformation Dil_{Id} ($p < 0.001$, 2-tailed test) and a significant effect for transformation Dil_{Pi} ($p < 0.01$, 2-tailed test).

(ii) the effects of the eccentricity of the large pattern with the small pattern eccentric (position combination aC compared with Aa and Ae): there is no significant effect for either transformation Dil_{Id} or Dil_{Pi} ($p > 0.1$, $p > 0.6$, respectively, 2-tailed tests).

(iii) the effect of changing both size and position (position combination Ac compared with aC): there is a significant effect for transformation Dil_{Id} ($p < 0.05$, 2-tailed test) and no significant effect for transformation Dil_{Pi} ($p > 0.05$, 2-tailed test).

(iv) the effects of eccentricity of the large pattern with the small pattern central (position combination Ac compared with Cc): there is no significant effect for either transformation Dil_{Id} or

Dil_{Pi} ($p > 0.05$, $p > 0.9$ respectively, 2-tailed tests)

(v) effects of size (position combinations AA and AE compared with aa and ae): there is no significant effect for transformation Id ($p > 0.1$, 2-tailed test), and a significant effect for transformation Pi ($p < 0.01$, 2-tailed test).

(vi) effects of the size of one of two eccentric patterns (position combinations Aa and Ae compared with aa and ae): there is no significant difference for transformations Dil_{Id} versus Id or Dil_{Pi} versus Pi ($p > 0.05$, $p > 0.2$ respectively, 2-tailed tests).

(vii) effects of distance and symmetry - (position combination Aa (aa or AA) compared with position combination Ae (ae or AE): there is a significant effect for transformations Dil_{Id}, Id (large patterns) and Id (small patterns) ($p < 0.05$, $p < 0.01$, $p < 0.01$ respectively, 2-tailed tests) and no significant effect for transformations Dil_{Pi}, Pi (large patterns) or Pi (small patterns) $p > 0.3$, $p > 0.9$, $p > 0.4$ respectively, 2-tailed test).

To summarize, performance on patterns related by dilatation is best when the smaller pattern is presented centrally, independent of the eccentricity of the large pattern. When the smaller pattern is presented eccentrically, performance is best when the positions of the two patterns coincide, and falls off with increasing pattern separation.

Performance for inverted dilatated patterns is also best when the smaller pattern is presented centrally, although this effect is not so marked, and did not show up in Experiment 7.1. Again, performance is independent of the eccentricity of the large pattern.

These results cannot be explained by suggesting that they are somehow caused by the small size of the patterns (see Section 7.1.2), since there is no difference between performance on the smaller and larger patterns for transformation Id.

7.3 Discussion

These results fail to fulfill the predictions of either of the schemes put forward at the start of this Chapter, nor are they naturally predicted by any other scheme cited here.

The results of Experiment 7.3 do suggest that size information is encoded separately from position information, or at least that position information is not scaled by size information, since modifying the global position component in the IR to represent '1° to the right of the fixation point' instead of '1° to the left of the fixation point' has the same cost in terms of discrimination for small and large patterns. In order to explain the relative importance of the position of the smaller pattern, however, one has to suggest ad hoc mechanisms. For example, one explanation would be that (a) the operation equivalent to magnification is easier than that for demagnification; (b) the operation equivalent to magnification can be applied only to IRs which represent patterns centred on the fixation point.

Since the ad hoc assumptions at the start of this Chapter are not confirmed by these results, the special role of size information in the IR and the effects of dilatations on this size information remain unclear. Further research is needed to resolve the issue of the way in which size information is encoded in the IR.

8. THE EFFECTS OF CERTAIN NON-RIGID TRANSFORMATIONS

If one assumes that stimuli are internally encoded by local features and the relations between them, a question of interest is whether the features in two IRs can be compared independently of the relations between them, and whether the relations can be compared independently of the features. The scheme proposed in Chapter 4 allows operations on the whole IR by modification of the meaning or interpretation of all elements of a given type in the IR; the model also allows operations on individual elements of the IR by continuous modification. It is implicit in the proposed scheme that in the IR, there are labels specifying 'sense' or orientation attached both to the local features and to the feature-relations between them; the global relabelling operation that can be used in the comparison of IRs must operate on *all* these labels, and cannot be applied to a subset of them. Hence, according to the scheme, it should not be possible to match patterns by the *identity* of local features alone, independent of the relations between them (but see Chapter 6), and it should not be possible to modify local features independent of the relations between them, except by (inefficient) use of the proposed continuous modification operation. The scheme can thus be seen to assume a fixed association between the features and relations in the IR. This assumption is partially justified by the fact that, for example, one cannot ignore position information when making comparisons between the IRs of identical stimuli at different positions (see Chapters 3 and 4). Also, Foster (1978) has shown that for pairs of same shape random dot patterns related by rotations, judgements of equality of dot number depend on the angle of rotation in much the same way as judgements of equality of shape (see Fig. 1.2). Foster assumes that local features, such as dot clusters characterized by dot density and cluster area, are invariant to rotation of the stimulus. Subjects do not appear to compare such features in the IR independently of the relations between them, since if they were doing so one would not expect any rotation dependence in the ability to judge equality of dot number.

An implication of the assumption that features and the relations between them are inextricably associated with one another is that there exist some transformations which can relate pairs of patterns and which, although apparently simple, should make the detection of 'sameness' of the pairs very difficult. Two such transformations are:

(i) inversion of all local features in a stimulus about their local centres (which will be labelled transformation Pi_F);

(ii) inversion of the positions of all local features about the centre of the stimulus, leaving the orientations of the features unchanged (which will be labelled transformation Pi_p).

If there exists an efficient way to match local features in the IR independent of their orientations (that is, if features do not have 'sense' labels) or if there exists an efficient way of inverting the 'sense' of all local features in the IR independent of the 'sense' of the relations between them (that is, if it is possible to operate on local features alone) then 'same' - detection performance on patterns related by transformation Pi_F should be high.

If there exists an efficient way of matching features of the same orientation, independent of position (that is, if it is possible to ignore feature-relations) or if there exists an efficient way of inverting the 'sense' of feature-position information without modifying the local features themselves (that is, if it is possible to operate on the relations, independent of the local features) then 'same'-

detection performance on patterns related by transformation Pi_P should be high.

If, as the scheme suggests, none of these operations exist, the only way in which these stimuli can be detected as 'same' is by use of the continuous modification operation; this is inefficient, and will make 'same' detection performance very poor.

The experiment reported here tests whether or not pairs of patterns related by these transformations are easily detected as 'same'. In order to be able to do this it must be possible to identify the features in the stimulus which are going to be encoded in the IR. This is achieved by generating the pattern-stimuli from a fixed set of subpatterns designated as local features. The local features chosen were an L shape and a T shape, each made of four dots, and chevrons of 140°, 90°, and 60° angle, made up of three dots each. All these 'local features' are rotationally asymmetric, so that the inverted 'local feature' is not identical to the original. Pairs of stimuli generated from these 'local features' were presented to the subject sequentially, each pair being related by one of four 'same' transformations: Id; Pi_F; Pi_P; Pi; or 'different'. The subject's task was to decide if the two patterns were 'same' taking the transformations (which were carefully explained to the subject) into account, or 'different'.

8.1 Experiment 8

8.1.1 Methods

Subjects. The subjects were five male students in the Department of

Communication and Neuroscience. All were aged between 22 and 27 years, had normal or corrected-to-normal vision, and all, except the author, were unaware of the purpose of the experiment.

The display. The display was as described in Section 2.1. Fixation was aided by four computer generated white lines, each about 0.25° long, pointing to the centre of the display, and by five dots forming a cross whose side was 1.25°. The lines were displayed throughout each presentation; the cross was extinguished at the start of each trial.

Stimuli. The stimuli were non-random dot patterns generated from a random selection, with replacement, of five of the 'features' shown in Fig. 8.1. These are '140°', '90°', '60°', 'T' and 'L', and were chosen to be reasonably representative of the features that might commonly occur in random dot patterns. The positions of the 'local features' were generated in the same way as the positions of the random dots in standard dot patterns (see Section 2.3), with a minimum centre-to-centre distance of 0.167°, and with all 'local feature' centres constrained to lie within an imaginary disc of 0.25° radius. The orientation of each 'local feature' was chosen randomly each time. Some examples of patterns so generated are shown in Fig. 8.2.

Pattern positions. In each trial two patterns appeared sequentially, centred on the fixation point.

Pattern transformations. There were four possible transformations relating the patterns in each 'same' pair. These were:

Id: the two patterns were identical;

Pi_F: one pattern was obtained from the other by inversion (that is, planar rotation through 180°) of the 'local features' about their *local* centres; the position of each feature was the same in both patterns;

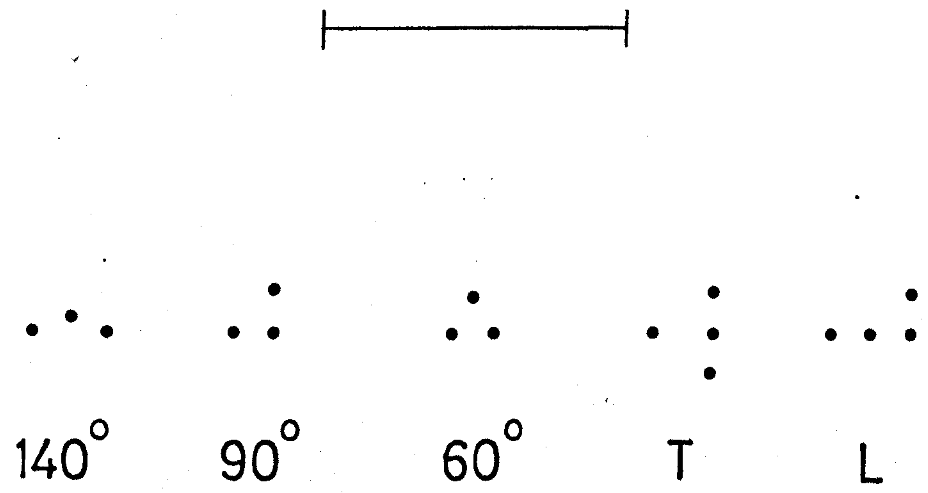


Fig 8.1

Figure 8.1. 'Features' used in the generation of the stimuli for Experiment 8. The scale marker represent 0.5° visual angle (dot diameter not to scale).

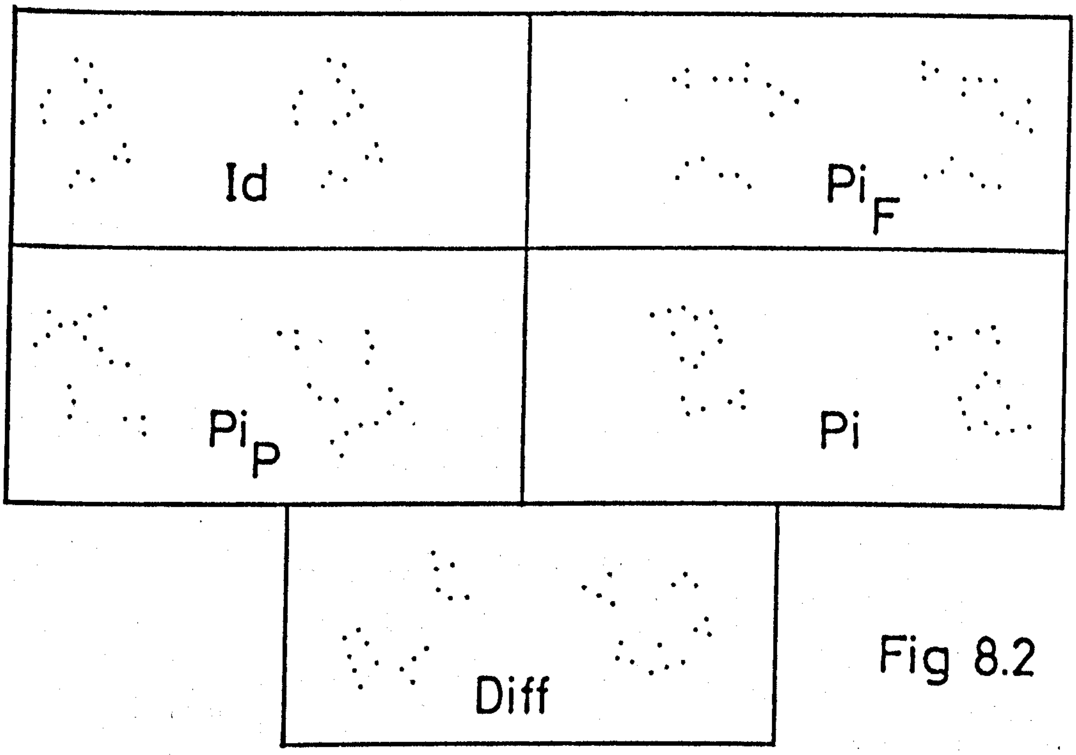


Fig 8.2

Figure 8.2. Random dot patterns used as stimuli in Experiment 8; examples of the transformations that could relate the patterns in a pair.

Pi_P: one pattern was obtained from the other by inversion of the *positions* of the 'features' about the centre of the *pattern*; the orientation of each 'feature' was the same for both patterns;

Pi: one pattern was obtained from the other by point inversion, that is, planar rotation of the whole pattern through 180° about the centre of the pattern. Note that transformation Pi is equivalent to the product of transformations Pi_F and Pi_P.

For 'different' pairs, two independent patterns were generated; that is, a new random selection of 'local features' was made, and the positions and orientations of the 'local features' were different. Examples of 'same' and 'different' pairs are illustrated in Fig. 8.2.

A fresh pattern or pair of patterns was generated for every trial.

Instructions and presentation sequence. The instructions to the subject and the sequence of events in a presentation were as described in Sections 2.4 and 2.5. The nature of the transformations involved was carefully explained to the subject and, before the start of the experiment, each subject was given a trial run to ensure that he understood the instructions.

Experimental design. In each run, every 'same' pattern transformation occurred five times; there were also twenty 'different' patterns in a run. Each subject performed a total of ten runs in one session. The sequence of the pattern transformations in a run was chosen randomly but balanced for order and carry-over effects over runs (see Section 2.6).

8.1.2 Results

Fig. 8.3 shows 'same-different' pattern discrimination performance

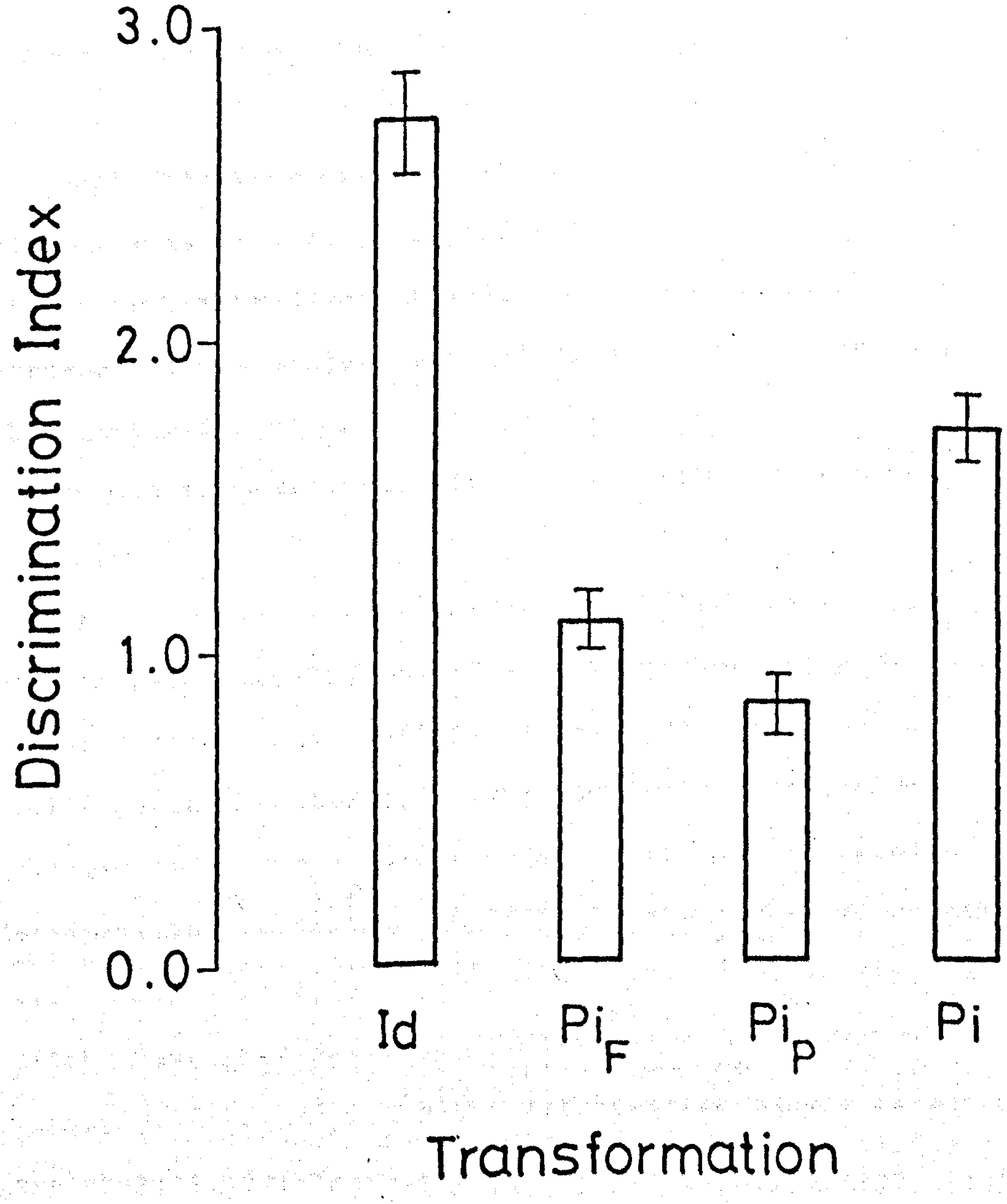


Fig 8.3

Figure 8.3. 'Same' detection performance in Experiment 8. The pooled discrimination index d' is shown for each of the transformations (Id, PI_F, PI_P, and PI).

in Experiment 8. The discrimination index d' (see Section 2.7) is plotted against pattern transformation.

The d' data are pooled over subjects. Chi-squared tests on individual data revealed (i) significant differences between subjects' absolute performance levels ($p < .001$) and (ii) no significant differences between subjects allowing for each subject's overall performance level ($p > 0.1$).

Contrast tests on the pooled data (see Section 2.7) yielded the following results:

(i) 'same' detection performance for patterns related by transformation Id is significantly higher than that for patterns related by transformation Pi ($p < 0.001$, 2-tailed test);

(ii) 'same' detection performance for patterns related by transformation Pi is significantly higher than that for patterns related by either of transformations Pi_F or Pi_P ($p < 0.001$ in both cases, 2-tailed tests);

(iii) there is no significant difference between 'same' detection performance on patterns related by transformation Pi_F and those related by transformation Pi_P ($p > 0.05$, 2-tailed test);

(iv) 'same' detection performance for patterns related either by Pi_F or Pi_P is significantly greater than chance level (zero d') ($p < 0.001$, in both cases, 2-tailed tests).

8.2 Discussion

We conclude from these results that it is not simple to invert the 'sense' of local features in the IR without modifying the structural parts of the IR, and vice versa. Performance for transformation Pi is higher than that for either of the non-rigid transformations, although performance for these is significantly greater than chance level. This suggests that it is possible to invert parts of IRs independently, although such operations have a high cost in terms of 'same' detection performance; this conclusion is consistent with the 'continuous modification' operation suggested in Section 4.4.

One objection to these results might be made on the grounds that possibly subjects were not able to identify the 'features' which were to be inverted, so that the transformations which define 'same' are not well specified. If this were true then pairs of patterns related by transformations Pi_P or Pi_F would be indistinguishable from 'different' pairs, and performance for these patterns would be at chance level, which is not the case. Note that, since the minimum centre-to-centre separation of 'features' is $0.167°$, and the radius of the disc within which each 'feature' must fall is $0.25°$, the distribution of the positions of the 'features' is fairly uniform. This means that the significantly non-zero performance levels on the non-rigid transformations cannot be explained by the suggestion that a proportion of the patterns by chance contained 'well separated' features, whereas the remainder of the patterns did not, and that only when well separated could the features be easily identified.

We conclude that there is a strong association between the features and the structure in the IR, but that the association is not completely 'fixed'; it is possible, although not easy, to operate on the features alone, or on the positions of the features alone. The global relabelling operation proposed in Section 4.4 cannot, however, be used to modify the 'sense' of local features independently of the relations in the IR, and vice versa.

9. SUMMARY

In this thesis, evidence has been presented concerning the ability of subjects to make 'same-different' judgements about pairs of patterns related by various transformations and arranged in the visual field in various ways. The evidence gives an insight into the way in which stimuli are represented in the visual system, and into the kinds of operations that can be performed on this internal representation. In this Chapter the lines of argument and evidence in the preceding Chapters will be summarized.

In Chapter 1 two types of scheme for the processing of visual stimuli were introduced: structural schemes and transformation schemes. The evidence presented in Chapter 1 favoured structural schemes rather than transformation schemes, although no final determination could be made.

It was suggested that the 'template' or pictorial type of internal representation (IR) usually put forward in transformation schemes takes no account of the structure of stimuli, and therefore cannot discard redundant information in the stimulus. In contrast, structural IRs discard much redundant stimulus information. Although transformation schemes predict 'mental rotation effects', it was suggested that such effects are a property of the cognitive tasks that are used to demonstrate them; structural schemes do not predict these effects but can explain the ability of observers to detect the 'sameness' of pairs of patterns related by rotation through 180°.

Other evidence was presented to suggest that the way in which stimuli are arranged in the visual field can have effects on the ability of subjects to perform various tasks. In particular pattern arrangement affects the ability to detect stimulus symmetry and to discriminate mirror image stimuli. It was suggested that these effects point to the significance of the existence of an axis of symmetry and to the significance of its orientation and position with respect to the point of fixation.

This evidence, which was not adequately dealt with by either transformation or structural schemes, suggested an experiment to systematically investigate the effects of two parameters of the stimulus arrangement, namely the symmetry of pattern positions with respect to the point of fixation, and the separation of pattern positions. In Chapter 3, evidence was presented about the way in which the effects of these parameters interact with the effects of four pattern transformations: identity, planar rotation through 90°, point inversion (that is, planar rotation through 180°), and reflection about a vertical axis. It was demonstrated that the ability to detect the 'sameness' of pairs of identical patterns depends mainly on the distance between the patterns, independent of the symmetry of the pattern positions with respect to the point of fixation, whereas the ability to detect the 'sameness' of reflected or point-inverted patterns depends mainly on the symmetry of the pattern positions, independent of their separation. The ability to detect the 'sameness' of 90° rotated patterns showed no clear dependence on either of these parameters, and was poorer than that for the other transformations.

It was argued in Chapter 4 that both transformation and structural schemes fail to explain the results. It was suggested that transformation schemes ought naturally to imply a monotonic dependence of 'same' detection performance on the 'size' of the transformation for which the system has to compensate. Observed performance showed two types of non-monotonicity: firstly, pattern separation affects performance for identical patterns, but does not affect performance for reflected or point-inverted patterns, and secondly, for pairs of patterns related by rotations, thre is a non-monotonic dependence of performance on rotation angle. Structural schemes do not naturally predict any of the observed effects apart from the relatively high performance on point-inverted patterns in one experimental condition.

For these reasons, a new scheme for the visual processing of patterns was proposed. In this scheme the IR is thought of as an extension of a structural IR to include elements representing the position of the stimulus with respect to the point of fixation. The relations encoded in the IR specify only horizontal or vertical relationships between features. Two types of permissible operation on this IR were proposed: a progressive continuous modification of individual elements in the IR, and a global relabelling of all the elements of a given type in the IR. Given the *a priori* evidence in favour of some form of structural representation, and the new data described above, it is suggested that this is the most parsimonious scheme available.

It was argued that the scheme can explain the results of Chapter 3, as follows:

(i) pairs of identical patterns are detected as 'same' by continuous modification of the position component in the IR of one of the patterns;

(ii) pairs of patterns related by point inversion are detected as 'same' by relabelling as their opposites all those elements specifying spatial direction or sense in the IR of one of the patterns; this operation brings the IRs of the two patterns into coincidence only if the positions of the patterns are symmetric with respect to the point of fixation;

(iii) pairs of patterns related by reflection in a vertical line are detected as 'same' by relabelling as their opposites all those elements specifying horizontal direction or sense in the IR of one of the patterns; again, this operation brings the IRs of the two patterns into coincidence only if the positions of the patterns are symmetric with respect to the point of fixation.

Some predictions of the model were developed, and the predictions were tested in the experiments described in Chapters 5 and 8. The predictions concerned the expected effects of pattern arrangement on the ability to detect the 'sameness' of reflected or point-inverted patterns, and the ability to detect the 'sameness' of patterns related by certain non-rigid transformations. The predictions concerning pattern arrangement can be summarized as follows:

(i) 'same' detection performance for side-by-side reflected patterns should be highest when the reflection is in a vertical line;

(ii) 'same' detection performance for reflected patterns should show a form of oblique effect (which should not be found for identical or point-inverted patterns);

(iii) 'same' detection performance for pairs of patterns related by reflection in a vertical line should not be affected by the vertical positions of the patterns, provided that these are the same for both patterns; this should also be true of identical patterns, but vertical positioning *should* affect 'same' detection performance for point-inverted patterns.

Three experiments testing these predictions were described in Chapter 5. The fact that all three predictions were fulfilled lends strong support to the main elements of the scheme, namely that the IR contains only horizontal and vertical relations; that these two sets of relations can be globally relabelled as their opposites either independently or jointly; that the IR contains relations encoding position information with respect to the point of fixation, and that this position information can be modified in a progressive continuous fashion.

The further predictions concerning non-rigid transformations were that 'same' detection performance for pairs of patterns related by point-inversion should be better than that for pairs related by either of the following transformations:

(i) inversion of local features about their local centres;

(ii) inversion of feature positions about the pattern centre without modification of the features themselves.

These predictions, and an experiment testing them, were described in Chapter 8; the results fulfilled the predictions. This finding makes it plausible that features in the IR cannot be compared independently of the relations between them and vice versa, so that it is reasonable to suppose, as is implicit in the proposed scheme, that comparisons of IRs cannot be performed on subsets of the features and feature-relations in the IRs; the whole IR must be taken into account.

Two further topics were considered: the detection of 'sameness' of rotated patterns, and the detection of 'sameness' of dilatated patterns.

Since the above scheme is not specifically equipped to respond to pairs of patterns related by rotations not close to 0° or 180°, some alternative explanation of the greater than chance level performance for these patterns had to be found. It was suggested that such patterns could be detected as 'same' only if they contained special 'strong' features, such as spurs, or clusters, and that 'strong' features could be compared directly, independently of the structure of the stimulus. Such a mechanism would be available for the comparison of all the 'same' stimuli, but in the case of identical, reflected or point-inverted patterns there is a more efficient structural mechanism available. It was therefore argued that 'same' detection performance for 90° rotated patterns should be strongly dependent on the pattern itself, whereas performance for patterns related by the other transformations should be either pattern independent or only weakly dependent on the pattern.

The results presented in Chapter 6 were consistent with this notion: the detection of 'sameness' of identical or point-inverted patterns is largely pattern independent; pattern specific effects are of significance only for the detection of 'sameness' of 90°-rotated patterns. It is concluded that pattern specific effects are of minor importance in the overall scheme.

The proposed scheme does not give rise to any obvious predictions concerning the ability to detect the 'sameness' of dilatated patterns. Two possible ways in which performance for such patterns might depend on pattern arrangement and pattern transformation were suggested in Chapter 7, neither of which turned out to be consistent with the experimental results presented in Chapter 7. It was suggested that further research is necessary to clarify the special role of size information in the internal representation.

In conclusion, the evidence presented in this thesis is strongly in favour of the notion that the internal representation is structural, and contains elements representing the local features in the stimulus, elements representing the horizontal and vertical relations between those features, and elements representing the position of the stimulus with respect to the point of fixation. The evidence presented here further supports the notion that there are two types of operation that can be performed on this representation: a progressive continuous modification of individual elements in the IR, and a global relabelling of all elements of a given type in the IR. These notions bring together the formerly disparate concepts of symbolic representations and dynamical operations, to form a scheme in which both discrete and continuous operations are possible.

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